Summer Solutions.

Minutes a Day–Mastery for a Lifetime!

Level 4

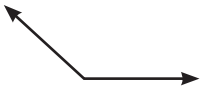
Problem Solving

Help Pages

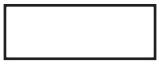
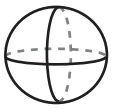
Help Pages

General Terms	
acute angle	an angle measuring less than 90° 
addend	a number being added
angle	formed by two rays that share a common endpoint
area	the size of a surface; always expressed in square units (inches^2, meters^2,...)
circumference	the distance around the outside of a circle
composite number	a number that has more than 2 factors
congruent	figures with the same shape and the same size
denominator	the bottom number of a fraction Example: $\frac{1}{4}$; the denominator is 4.
diameter	the widest distance across a circle; the diameter always passes through the center
equation	a math "sentence" that uses numbers, math symbols, and an "=" sign
difference	the result or answer to a subtraction problem Example: the difference of 5 and 1 is 4.
factor	a whole number that can be divided into a given number without a remainder
fraction	a part of a whole Example:  This box has 4 parts; 1 part is shaded. $\frac{1}{4}$ is shaded.
improper fraction	a fraction in which the numerator is larger than the denominator Example: $\frac{9}{4}$
line of symmetry	a line along which a figure can be folded so that the two halves match exactly
mixed number	the sum of a whole number and a fraction Example: $5\frac{1}{4}$

Help Pages

General Terms	
numerator	the top number of a fraction Example: $\frac{1}{4}$; the numerator is 1.
obtuse angle	an angle measuring more than 90° 
perimeter	the distance around the outside of a polygon
prime number	a number that has only 2 factors, one and itself
product	the result or answer to a multiplication problem Example: The product of 5 and 3 is 15.
quotient	the result or answer to a division problem Example: The quotient of 8 and 2 is 4.
radius	the distance from any point on the circle to the center; half of the diameter
ray	a line that has a starting point, but no endpoint
represent	show
right angle	an angle measuring exactly 90° 
similar	figures having the same shape, but different sizes
straight angle	an angle measuring exactly 180°
sum	the result or answer to an addition problem Example: The sum of 5 and 2 is 7.
volume	the measure of space inside a solid figure; always expressed in cubic units (m^3, ft^3,...)

Help Pages

2-Dimensional Shapes			
circle		ellipse	 (oval)
triangle	 any shape with 3 sides	quadrilateral	 any shape with 4 sides
parallelogram		rectangle	
square		rhombus	 (diamond)
trapezoid		pentagon	 any shape with 5 sides
hexagon	 any shape with 6 sides	octagon	 any shape with 8 sides
3-Dimensional Shapes			
pyramid		cone	
rectangular prism		cube	
sphere		cylinder	

Help Pages

Measurement — Relationships

Volume

8 ounces = 1 cup

3 teaspoons = 1 tablespoon

2 cups = 1 pint

2 pints = 1 quart

4 quarts = 1 gallon

Distance

12 inches = 1 foot

36 inches = 1 yard

1,760 yards = 1 mile

5,280 feet = 1 mile

100 centimeters = 1 meter

1,000 millimeters = 1 meter

Weight

16 ounces = 1 pound

2,000 pounds = 1 ton

Time

10 years = 1 decade

100 years = 1 century

Temperature

0° Celsius → freezing point of water

100° Celsius → boiling point of water

32° Fahrenheit → freezing point of water

212° Fahrenheit → boiling point of water

Place Value — Whole Numbers

8,	9	6	3,	2	7	1,	4	0	5
Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones

The number above is read: eight billion, nine hundred sixty-three million, two hundred seventy-one thousand, four hundred five.

Place Value — Decimal Numbers

1	7	8	.	6	4	0	5	9	2
Hundreds	Tens	Ones	*Decimal Point* →	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths

The number above is read: one hundred seventy-eight and six hundred forty thousand, five hundred ninety-two millionths.

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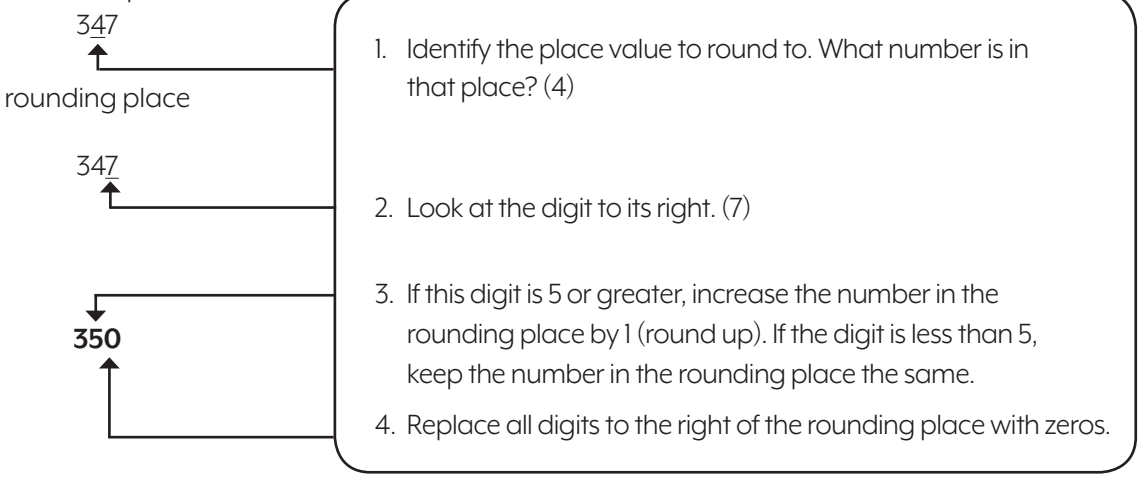
Whole Numbers

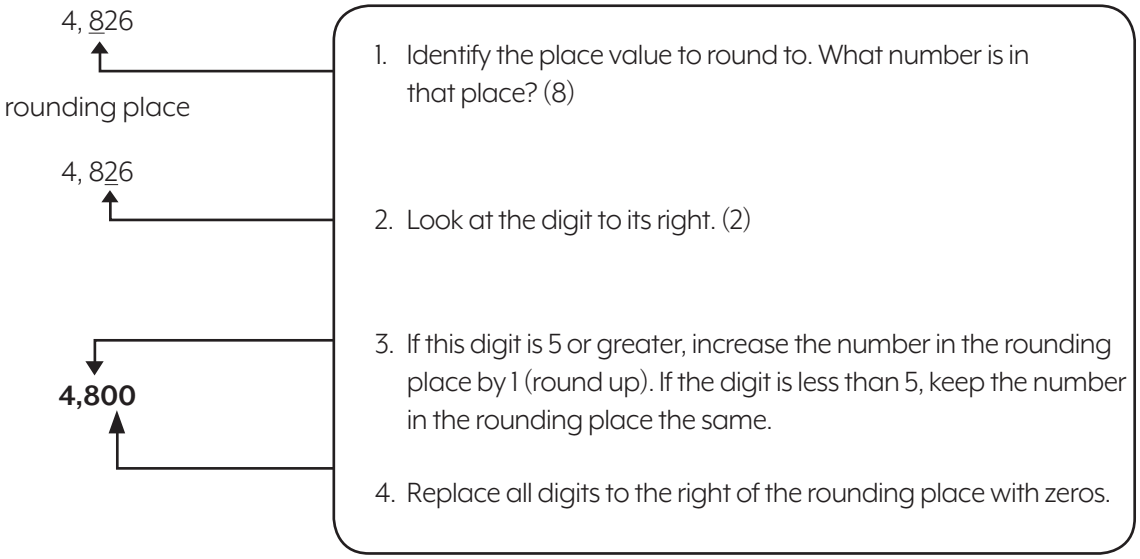
Think of **rounding numbers** as an easier way to work with numbers. Rounding is a way of estimating. The rounded number (or estimate) is close to the actual value, but has zeros at the end. Use a place value chart if needed.

Examples:

Round 347 to the tens place.



Round 4,826 to the hundreds place.



Round 27,934 to the thousands place.

27,934	7 is in the rounding place.
27,934	9 is greater than 5, so the rounding place will go up by 1.
28,000	The digits to the right of the rounding place are changed to zeros.

Help Pages

Whole Numbers (continued)

When **adding or subtracting whole numbers**, first the numbers must be lined-up from the right. Starting with the ones place, add (or subtract) the numbers. When adding, if the answer has two digits, write the ones digit and regroup the tens digit. For subtraction, it may also be necessary to regroup first. Then, add (or subtract) the numbers in the tens place. Continue with the hundreds, etc.

Look at these **addition** examples.

Find the sum of 314 and 12.

$$\begin{array}{r} 314 \\ +\ 12 \\ \hline 326 \end{array}$$

1. Line up the numbers on the right.
2. Beginning with the ones place, add. Regroup if necessary.
3. Repeat with the tens place.
4. Continue this process with the hundreds place, etc.

Add 6,478 and 1,843.

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 6{,}478 \\ +1{,}843 \\ \hline 8{,}321 \end{array}$$

Look at these **subtraction** examples.

Subtract 37 from 93.

$$\begin{array}{r} {\scriptstyle 8\ 13} \\ 9\not{3} \\ -\ 37 \\ \hline 56 \end{array}$$

1. Begin with the ones place.
   Since 7 is larger than 3, regroup to 8 tens and 13 ones.
2. Now look at the tens place.
   Since 3 is less than 8, the regrouping is complete.
3. Subtract each place value beginning with the ones.

Find the difference of 4,125 and 2,033.

$$\begin{array}{r} {\scriptstyle 0\ 12} \\ 4{,}1\not{2}5 \\ -\ 2{,}033 \\ \hline 2{,}092 \end{array}$$

1. Begin with the ones place. Since 3 is less than 5, do not regroup.
2. Now look at the tens place.
   Since 3 is larger than 2, regroup to 0 hundreds and 12 tens.
3. Now look at the hundreds place.
   Since 0 can be taken from 0, do not regroup.
4. Now look at the thousands place. Since 2 is smaller than 4, do not regroup.
5. Subtract each place value beginning with the ones.

When **subtracting from zero**, always regroup.

Subtract 2,361 from 5,000.

$$\begin{array}{r} {\scriptstyle 4\ 10\ 10\ 10} \\ 5{,}\not{0}\not{0}\not{0} \\ -\ 2{,}361 \\ \hline 2{,}639 \end{array}$$

1. Begin with the ones place. Since 1 is larger than 0, regroup.
   Continue to the thousands place, and then begin regrouping.
2. Regroup the thousands place to 4 thousand and 10 hundreds.
3. Next, regroup the hundreds place to 9 hundreds and 10 tens.
4. Then, regroup the tens place to 9 tens and 10 ones.
5. Finally, subtract each place value beginning with the ones.

Example: Find the difference between 600 and 238.

$$\begin{array}{r} {\scriptstyle 5\ 10\ 10} \\ 6\not{0}\not{0} \\ -\ 238 \\ \hline 362 \end{array}$$

Help Pages

Whole Numbers (continued)

When **multiplying multi-digit whole numbers**, it is important to know the multiplication facts. Follow the steps and the examples below.

Examples:

Multiply 23 by 5.

1. Line up the numbers on the right.
2. Multiply the digits in the ones place. Regroup if necessary.
3. Multiply the digits in the tens place. Add any regrouped tens.
4. Repeat step 3 for the hundreds place, etc.

$$\begin{array}{r} {\scriptstyle 1}\,23 \\ \times\ \ 5 \\ \hline 115 \end{array}$$

3 × 5 = 15 ones or 1 ten and 5 ones
2 × 5 = 10 tens + 1 ten (regrouped) or 11 tens

Find the product of 3,514 and 3.

$$\begin{array}{r} {\scriptstyle 1\ \ 1}\,3{,}5\,1\,4 \\ \times\ \ \ \ \ 3 \\ \hline 10{,}542 \end{array}$$

4 × 3 = 12 ones or 1 ten and 2 ones
1 × 3 = 3 tens + 1 ten (regrouped) or 4 tens
5 × 3 = 15 hundreds or 1 thousand and 5 hundreds
3 × 3 = 9 thousands + 1 thousand (regrouped) or 10 thousands

The process for **multiplying by two-digit numbers** is a lot like the process above. There are a few differences. Follow the steps carefully.

Examples:

Multiply 32 by 24.

$$\begin{array}{r} 32 \\ \times\ 24 \\ \hline 128 \leftarrow ❶ \\ 0 \leftarrow ❷ \end{array}$$

$$\begin{array}{r} 32 \\ \times\ 24 \\ \hline 128 \\ +\ 640 \leftarrow ❸ \\ \hline 768 \leftarrow ❹ \end{array}$$

1. Multiply each digit in the top number by the ones digit in the bottom number. Regroup if necessary. (4 × 2 = 8; 4 × 3 = 12)
2. When working with the tens digit, the answer will be written below the previous answer. Before multiplying by the tens digit, put a zero in the ones place.
3. Multiply each digit in the top number by the tens digit in the bottom number. Regroup if necessary. (2 × 2 = 4; 2 × 3 = 6)
4. Add the products.

Find the product of 45 and 38.

$$\begin{array}{r} {\scriptstyle 4}\,45 \leftarrow ❶ \\ \times\ 38 \\ \hline 360 \\ 0 \leftarrow ❷ \end{array}$$

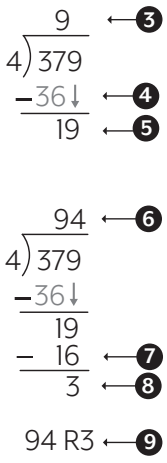
$$\begin{array}{r} {\scriptstyle 14}\,45 \leftarrow ❸ \\ \times\ 38 \\ \hline 360 \\ +\ 1{,}350 \\ \hline 1{,}710 \leftarrow ❹ \end{array}$$

Help Pages

Whole Numbers (continued)

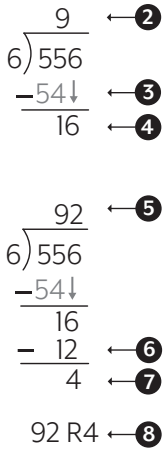
The next group of examples involves **long division using one-digit divisors with remainders**. This process, called "long division," will be used to divide numbers with multiple digits.

Example: Divide 379 by 4.



1. In this problem, 379 is the dividend, and 4 is the divisor. Look at each digit in the dividend, starting on the left.
2. Does the divisor (4) go into the left-most digit in the dividend (3)? It doesn't, so keep going to the right.
3. Does the divisor (4) go into the two left-most digits (37)? It does. How many times does 4 go into 37? (9 times)
4. Multiply 4 × 9. (36)
5. Subtract 36 from 37. (1) Bring down the 9 ones from the first line. This leaves 19 left from the original 379.
6. Does the divisor (4) go into 19? It does. How many times does 4 go into 19? (4 times)
7. Multiply 4 × 4. (16)
8. Subtract 16 from 19. (3) There's nothing left to bring down from above. Once this number is smaller than the divisor, it is called the remainder. The problem is finished. The remainder is 3.
9. Write the answer with the remainder.

Example: What is 556 divided by 6?



1. Does the divisor (6) go into the left-most digit in the dividend? (5) It doesn't, so keep going to the right.
2. Does the divisor (6) go into the two left-most digits? (55) It does. How many times does 6 go into 55? (9 times)
3. Multiply 6 × 9. (54)
4. Subtract 54 from 55. (1) Bring down the 6 ones from the first line. This leaves 16 left from the original 556.
5. Does the divisor (6) go into 16? It does. How many times does 6 go into 16? (2)
6. Multiply 6 × 2. (12)
7. Subtract 12 from 16. (4) There's nothing left to bring down from above. Once this number is smaller than the divisor, it is called the remainder. The problem is finished. The remainder is 4.
8. Write the answer with the remainder. (92 R 4)

Remember: The remainder can NEVER be larger than the divisor!

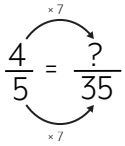
Help Pages

Fractions

Equivalent Fractions are 2 fractions that are equal to each other. Problems often ask for a missing numerator or denominator.

Examples:

Find a fraction that is equivalent to $\frac{4}{5}$ and has a denominator of 35.



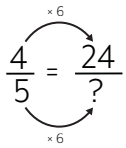
1. Ask, "What was done to 5 to get 35?" (Multiply by 7.)
2. Whatever is done in the denominator, must be done in the numerator.

   4 x 7 = 28

   The missing numerator is 28.

   So, $\frac{4}{5}$ is equivalent to $\frac{28}{35}$.

Example: Find a fraction that is equivalent to $\frac{4}{5}$ and has a numerator of 24.



1. Ask, "What was done to 4 to get 24?" (Multiply by 6.)
2. Whatever is done in the numerator, must be done in the denominator.

   5 x 6 = 30

   The missing denominator is 30.

   So, $\frac{4}{5}$ is equivalent to $\frac{24}{30}$.

To **add (or subtract) fractions with the same denominator**, simply add (or subtract) the numerators, keeping the same denominator.

Examples: $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

$\frac{8}{9} - \frac{1}{9} = \frac{7}{9}$

To **add mixed numbers**, follow a process similar to the one used with fractions. If the sum is an improper fraction, be sure to simplify it.

Example:
$$\begin{array}{r} 1\frac{2}{5} \\ +\,1\frac{4}{5} \\ \hline 2\frac{6}{5} \end{array}$$

$2\frac{6}{5}$ is improper. $\frac{6}{5}$ can be rewritten as $1\frac{1}{5}$.

So, $2\frac{6}{5}$ is $2 + 1\frac{1}{5} = 3\frac{1}{5}$.

Help Pages

Decimals

Adding and subtracting decimals is similar to adding and subtracting whole numbers. Lining up the decimal points of the number values is always the first step. Add zeros if necessary, so that all of the numbers have the same number of digits after the decimal point. The zeros don't change the value. Before subtracting, remember to regroup also. After adding or subtracting the number values, bring the decimal point straight down into the answer.

Examples:

Find the sum of 4.25 and 2.31.

$$\begin{array}{r} 4.25 \\ +\ 2.31 \\ \hline 6.56 \end{array}$$

1. Line up the decimal points. Add zeros as needed.
2. Add (or subtract) the decimals.
3. Add (or subtract) the whole numbers.
4. Bring the decimal point straight down.

Add 55.2 and 6.472.

$$\begin{array}{r} {\scriptstyle 1} \\ 55.200 \\ +\ 6.472 \\ \hline 61.672 \end{array}$$

Subtract 4.8 from 7.4.

$$\begin{array}{r} {\scriptstyle 6\ 14} \\ \not{7}.\not{4} \\ -\ 4.8 \\ \hline 2.6 \end{array}$$

Find the difference of 4.1 and 2.88.

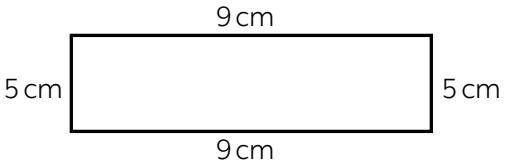
$$\begin{array}{r} {\scriptstyle 3\ 11\ 10} \\ \not{4}.\not{1}\not{0} \\ -\ 2.88 \\ \hline 1.22 \end{array}$$

Geometry

The **perimeter of a polygon** is the distance around the outside of the figure. To find the perimeter, add the lengths of the sides of the figure. Be sure to label the answer.

Perimeter = sum of the sides

Example: Find the perimeter of the rectangle below.



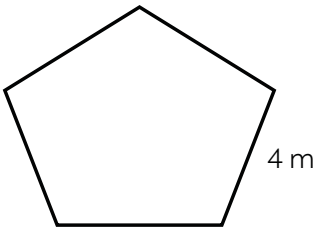
Perimeter = 5 cm + 9 cm + 5 cm + 9 cm

Perimeter = 28 cm

Help Pages

Geometry (continued)

Example: Find the perimeter of the regular pentagon below.



A pentagon has 5 sides. Each of the sides is 4 m long.

P = 4 m + 4 m + 4 m + 4 m + 4 m

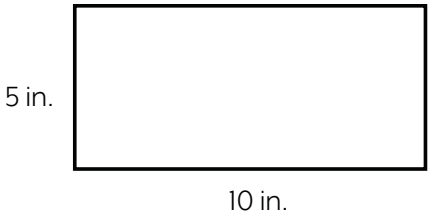
P = 5 × 4 m

P = 20 m

Area is the size of a surface. To find the area of a rectangle or a square, multiply the length by the width. The area is expressed in square units (ft^2, $in.^2$, etc.).

Area of rectangle = length × width or $A = l \times w$

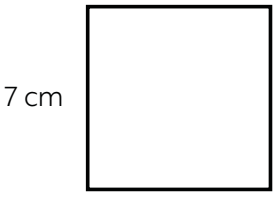
Examples: Find the area of the figures below.



Area = length × width

A = 10 in. × 5 in.

A = 50 $in.^2$ → Say "50 square inches."



A square has 4 equal sides, so its length and its width are the same.

A = 7 cm × 7 cm

A = 49 cm^2

Help Pages

Problem Solving Strategies

Make an Organized List

Some math problems ask for a list of all possible correct answers. This strategy helps you organize all of your ideas without repeating any answers.

Guess and Check

Some math problems ask you to think like a detective. Detectives follow clues to solve a "case." Guess and check as you work with one clue at a time. When the final answer fits every clue, you have solved the case.

Look for a Pattern

Some math problems ask you to write what comes next. In a pattern, numbers go in order according to a rule. The numbers in a pattern may be getting larger or smaller. This strategy helps you think about the rule a pattern is following.

Draw a Picture

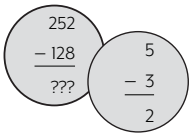
Some math problems are easier to understand through pictures. Draw a picture to act out the problem on paper.

Work Backward

Some math problems tell you the end of a story. Your task is to discover the beginning of the story. To use this strategy start with the answer and do the math steps in reverse.

Solve a Simpler Problem

Some math problems have numbers that seem too big. This strategy helps you find a basic fact you already know. You can use what you know to tackle the bigger numbers.

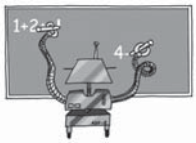


Use a Table/Make a Table

Some math problems give lots of information. Tables have rows, columns, and labels. A table helps you organize the information and see patterns.

Write a Number Sentence

Word problems can become numbers and math symbols (+ – ÷ × = < >). These numbers and math signs help you solve the problem.



Make a Model

Some math problems describe a scene that you begin to imagine. Make a model to help you act out the problem with objects.

Use Logical Reasoning

Some math problems are like puzzles. *If* this piece goes here, *then* this other piece must go there. Use logic to work in little bits until you see the whole answer.