Summer Solutions.

Minutes a Day–Mastery for a Lifetime!

Intermediate A

Mathematics

3rd Edition

Help Pages

Help Pages

Vocabulary

Arithmetic Operations
sum — the result or answer to an addition problem. Example: The sum of 5 and 2 is 7.
difference — the result or answer to a subtraction problem. Example: The difference of 5 and 1 is 4.
product — the result or answer to a multiplication problem. Example: The product of 5 and 3 is 15.
quotient — the result or answer to a division problem. Example: The quotient of 8 and 2 is 4.
Factors and Multiples
factors — are multiplied together to get a product. Example: 2 and 3 are factors of 6.
multiples — can be evenly divided by a number. Example: 5, 10, 15, and 20 are multiples of 5.
composite number — a number with more than 2 factors. Example: 10 has factors of 1, 2, 5, and 10. Ten is a composite number.
prime number — a number with exactly 2 factors (the number itself and 1). Example: 7 has factors of 1 and 7. Seven is a prime number.
greatest common factor (GCF) — the highest factor that 2 numbers have in common. Example: The factors of 6 are 1, 2, 3, and 6. The factors of 9 are 1, 3, and 9. The GCF of 6 and 9 is 3.
least common multiple (LCM) — the smallest multiple that 2 numbers have in common. Example: Multiples of 3 are 3, 6, 9, 12, 15… Multiples of 4 are 4, 8, 12, 16… The LCM of 3 and 4 is 12.
prime factorization — a number, written as a product of its prime factors. Example: 140 can be written as 2 x 2 x 5 x 7. (All of these are prime factors of 140.)
Fractions and Decimals
improper fraction — a fraction in which the numerator is larger than the denominator. Example: $\frac{9}{4}$
mixed number — a whole number and a fraction. Example: $5\frac{1}{4}$
reciprocal — a fraction in which the numerator and denominator are interchanged. The product of a fraction and its reciprocal is always 1. Example: The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$. $\frac{3}{5} \times \frac{5}{3} = \frac{15}{15} = 1$
repeating decimal — a decimal in which a number or a series of numbers continues on and on. Example: 2.33333333, 4.151515151515, 7.125555555, etc.
Geometry
acute angle — an angle measuring less than 90°.
congruent — figures with the same shape and the same size.
obtuse angle — an angle measuring more than 90°.
right angle — an angle measuring exactly 90°.
similar — figures having the same shape, but different size.
straight angle — an angle measuring exactly 180°.

Help Pages

Vocabulary (continued)

Geometry — Circles

circumference — the distance around the outside of a circle.

diameter — the widest distance across a circle. The diameter always passes through the center.

radius — the distance from any point on the circle to the center. The radius is half of the diameter.

Geometry — Polygons

Number of Sides	Name	Number of Sides	Name
3	**triangle**	7	**heptagon**
4	**quadrilateral**	8	**octagon**
5	**pentagon**	9	**nonagon**
6	**hexagon**	10	**decagon**

Geometry — Triangles

equilateral — a triangle in which all 3 sides have the same length.

isosceles — a triangle in which 2 sides have the same length.

scalene — a triangle in which no sides are the same length.

Measurement — Relationships

Volume	Distance
3 teaspoons in a tablespoon	36 inches in a yard
2 cups in a pint	1,760 yards in a mile
2 pints in a quart	5,280 feet in a mile
4 quarts in a gallon	100 centimeters in a meter
Weight	1,000 millimeters in a meter
16 ounces in a pound	**Temperature**
2,000 pounds in a ton	0° Celsius – freezing point
Time	100°Celsius – boiling point
10 years in a decade	32°Fahrenheit – freezing point
100 years in a century	212°Fahrenheit – boiling point

Ratio and Proportion

proportion — a statement that two ratios (or fractions) are equal. Example: $\frac{1}{2} = \frac{3}{6}$

ratio — a comparison of two numbers by division; a ratio looks like a fraction. Example: $\frac{2}{5}$

Help Pages

Vocabulary (continued)

Statistics

Mean — the average of a group of numbers. The mean is found by finding the sum of a group of numbers and then dividing the sum by the number of members in the group.

Example: The average of 12, 18, 26, 17 and 22 is **19**. $\frac{12+18+26+17+22}{5} = \frac{95}{5} = 19$

Median — the middle value in a group of numbers. The median is found by listing the numbers in order from least to greatest, and finding the one that is in the middle of the list. If there is an even number of members in the group, the median is the average of the two middle numbers.

Example: The median of 14, 17, 24, 11 and 26 is **17**. 11, 14, (17), 24, 26

The median of 77, 93, 85, 95, 70 and 81 is **83**. 70, 77, (81, 85), 93, 95 $\frac{81+85}{2} = 83$

Mode — the number that occurs most often in a group of numbers. The mode is found by counting how many times each number occurs in the list. The number that occurs more than any other is the mode. Some groups of numbers have more than one mode.

Example: The mode of 77, (93), 85, (93), 77, 81, (93) and 71 is **93**. (93 occurs more than the others.)

Place Value

Whole Numbers

8,	9	6	3,	2	7	1,	4	0	5
Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones

The number above is read: eight billion, nine hundred sixty-three million, two hundred seventy-one thousand, four hundred five.

Decimal Numbers

1	7	8	.	6	4	0	5	9	2
Hundreds	Tens	Ones	*Decimal Point*	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths

The number above is read: one hundred seventy-eight and six hundred forty thousand, five hundred ninety-two millionths.

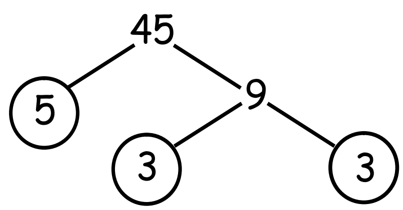
Help Pages

Solved Examples

Factors and Multiples

The **Prime Factorization** of a number is written as a product of its prime factors. A factor tree is helpful in finding the prime factors of a number.

Example: Use a factor tree to find the prime factors of 45.



1. Find any 2 factors of 45 (5 and 9).
2. If a factor is prime, circle it. If a factor is not prime, find 2 factors of it.
3. Continue until all factors are prime.
4. In the final answer, the prime factors are listed in order, least to greatest, using exponents when needed.

The prime factorization of 45 is $3 \times 3 \times 5$ or $3^2 \times 5$.

The **Greatest Common Factor (GCF)** is the largest factor that 2 numbers have in common.

Example: Find the Greatest Common Factor of 32 and 40.

The factors of 32 are 1, 2, 4, (8), 16, 32.

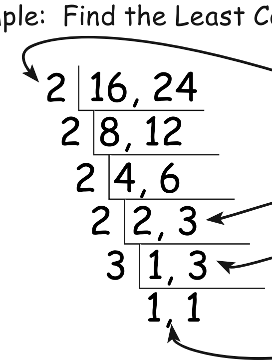
The factors of 40 are 1, 2, 4, 5, (8), 10, 20, 40.

The GCF of 32 and 40 is **8**.

1. First list the factors of each number.
2. Find the largest number that is in both lists.

The **Least Common Multiple (LCM)** is the smallest multiple that two numbers have in common. The prime factors of the numbers can be useful in finding the LCM.

Example: Find the Least Common Multiple of 16 and 24.



1. If any of the numbers are even, factor out a 2.
2. Continue factoring out 2 until all numbers left are odd.
3. If the prime number cannot be divided evenly into the number, simply bring the number down.
4. Once there are only all 1s at the bottom, it's finished!
5. Multiply all of the prime numbers (on the left side of the bracket) together to find the Least Common Multiple.

The LCM of 16 and 24 is $2 \times 2 \times 2 \times 2 \times 3$ or **48**.

Fractions

Changing from an improper fraction to a mixed number.

Example: Change the improper fraction, $\frac{5}{2}$, to a mixed number.

$\frac{5}{2}$ (five halves) means $5 \div 2$.

$$\begin{array}{r} 2 \\ 2\overline{)5} \\ -4 \\ \hline 1 \end{array}$$

So, $\frac{5}{2}$ is equal to 2 wholes and 1 half or $2\frac{1}{2}$.

Help Pages

Solved Examples

Fractions (continued)

Changing from a mixed number to an improper fraction.

Example: Change the mixed number, $7\frac{1}{4}$, to an improper fraction.

1. To find the numerator of the new fraction, multiply the whole number by the denominator, and add the numerator
2. Keep the same denominator in the new fraction that was in the mixed number.

$7\frac{1}{4}$ 7 x 4 = 28. 28 + 1 = 29.

The new numerator is 29.

Keep the same denominator, 4.

The new fraction is $\frac{29}{4}$.

$7\frac{1}{4}$ is equal to $\frac{29}{4}$.

Equivalent Fractions are 2 fractions that are equal to each other. Usually, the task is to find a missing numerator or denominator.

Example: Find a fraction that is equivalent to $\frac{4}{5}$ and has a denominator of 35.

x 7

$\frac{4}{5} = \frac{?}{35}$

x 7

1. Ask, "What was done to 5 to get 35?" (Multiply by 7.)
2. Whatever is done to the denominator must be done to the numerator. 4 x 7 = 28. The missing numerator is 28.

So, $\frac{4}{5}$ is equivalent to $\frac{28}{35}$.

Example: Find a fraction that is equivalent to $\frac{4}{5}$ and has a numerator of 24.

x 6

$\frac{4}{5} = \frac{24}{?}$

x 6

1. Ask, "What was done to 4 to get 24?" (Multiply by 6.)
2. Whatever is done to the numerator must be done to the denominator. 5 x 6 = 30. The missing denominator is 30.

So, $\frac{4}{5}$ is equivalent to $\frac{24}{30}$.

Comparing Fractions means looking at 2 or more fractions and determining if they are equal, if one is greater than (>) the other, or if one is less than (<) the other. A simple way to compare fractions is by cross-multiplying, using the steps below.

Examples: Compare these fractions. Use the correct symbol. $\frac{8}{9} \bigcirc \frac{3}{4}$ $\frac{7}{9} \bigcirc \frac{6}{7}$

32 27

$\frac{8}{9} \otimes \frac{3}{4}$

49 54

$\frac{7}{9} \otimes \frac{6}{7}$

So, $\frac{8}{9} > \frac{3}{4}$ and $\frac{7}{9} < \frac{6}{7}$.

1. Begin with the denominator on the left and multiply by the opposite numerator. Put the answer (product) above the side where you ended. (9 x 3 = 27)
2. Cross-multiply the other denominator and numerator and put that product above where you ended.
3. Compare the two products and insert the correct symbol.

   HINT: **Always** multiply diagonally **upwards**!

Help Pages

Solved Examples

Fractions (continued)

To **add (or subtract) fractions with the same denominator**, simply add (or subtract) the numerators, keeping the same denominator.

Examples: $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

$\frac{8}{9} - \frac{1}{9} = \frac{7}{9}$

To **add mixed numbers**, follow a process similar to the one you used with fractions. If the sum is an improper fraction, be sure to simplify it.

Example:
$$\begin{array}{r} 1\frac{2}{5} \\ +1\frac{4}{5} \\ \hline 2\frac{6}{5} \end{array}$$

$2\frac{6}{5}$ is improper. $\frac{6}{5}$ can be rewritten as $1\frac{1}{5}$.

So, $2\frac{6}{5}$ is $2 + 1\frac{1}{5} = 3\frac{1}{5}$.

When **adding fractions that have different denominators**, change the fractions so they have a common denominator before they can be added.

Finding the **Least Common Denominator (LCD)**:

The LCD of the fractions is the same as the Least Common Multiple of the denominators. Sometimes, the LCD will be the product of the denominators.

Example: Find the sum of $\frac{3}{8}$ and $\frac{1}{12}$.

$$\begin{array}{r} \frac{3}{8} = \frac{9}{24} \\ +\frac{1}{12} = \frac{2}{24} \\ \hline \frac{11}{24} \end{array}$$

1. First, find the LCM of 8 and 12.
2. The LCM of 8 and 12 is 24. This is also the LCD of these 2 fractions.
3. Find an equivalent fraction for each that has a denominator of 24.
4. When they have a common denominator, the fractions can be added.

$$\begin{array}{r|l} 2 & 8, 12 \\ 2 & 4, 6 \\ 2 & 2, 3 \\ 3 & 1, 3 \\ & 1, 1 \end{array}$$

$2 \times 2 \times 2 \times 3 = 24$

The LCM is 24.

So, the LCD is 24.

Example: Add $\frac{1}{4}$ and $\frac{1}{5}$.

$$\begin{array}{r} \frac{1}{4} = \frac{5}{20} \\ +\frac{1}{5} = \frac{4}{20} \\ \hline \frac{9}{20} \end{array}$$

$4 \times 5 = 20$ The LCM is 20. Use 20 as the LCD.

When **adding mixed numbers with unlike denominators**, follow a process similar to the one used with the fractions (above). Be sure to put the answer in simplest form.

Example: Find the sum of $6\frac{3}{7}$ and $5\frac{2}{3}$.

$$\begin{array}{r} 6\frac{3}{7} = 6\frac{9}{21} \\ +5\frac{2}{3} = 5\frac{14}{21} \\ \hline 11\frac{23}{21} \end{array}$$

$11\frac{23}{21}$ (improper)

$\frac{23}{21} = 1\frac{2}{21} + 11 = 12\frac{2}{21}$

1. Find the LCD.
2. Find the missing numerators.
3. Add the whole numbers, then add the fractions.
4. Make sure the answer is in simplest form.

Help Pages

Solved Examples

Fractions (continued)

When subtracting numbers with unlike denominators, follow a process similar to the one used when adding fractions. Be sure to put the answer in simplest form.

Examples: Find the difference of $\frac{3}{4}$ and $\frac{2}{5}$.

$$\begin{array}{r} \frac{3}{4} = \frac{15}{20} \\ -\frac{2}{5} = \frac{8}{20} \\ \hline \frac{7}{20} \end{array}$$

1. Find the LCD just as was done when adding fractions.
2. Find the missing numerators.
3. Subtract the numerators and keep the common denominator.
4. Make sure the answer is in simplest form.

Subtract $\frac{1}{16}$ from $\frac{3}{8}$.

$$\begin{array}{r} \frac{3}{8} = \frac{6}{16} \\ -\frac{1}{16} = \frac{1}{16} \\ \hline \frac{5}{16} \end{array}$$

When **subtracting mixed numbers with unlike denominators**, follow a process similar to the one used when adding mixed numbers. Be sure to put the answer in simplest form.

Example: Subtract $4\frac{2}{5}$ from $8\frac{9}{10}$.

1. Find the LCD.
2. Find the missing numerators.
3. Subtract and simplify the answer.

$$\begin{array}{r} 8\frac{9}{10} = 8\frac{9}{10} \\ -4\frac{2}{5} = 4\frac{4}{10} \\ \hline 4\frac{5}{10} = 4\frac{1}{2} \end{array}$$

Sometimes when subtracting mixed numbers, regrouping is needed. If the numerator of the top fraction is smaller than the numerator of the bottom fraction, borrow from the whole number.

Example: Subtract $5\frac{5}{6}$ from $9\frac{1}{4}$.

1. Find the LCD.
2. Find the missing numerators.
3. Because 10 can't be subtracted from 3, you need to borrow from the whole number.
4. Rename the whole number as a mixed number using the common denominator.
5. Add the 2 fractions to get an improper fraction.
6. Subtract the whole numbers and the fractions and simplify the answer.

$$\begin{array}{r} 9\frac{1}{4} = 9\frac{3}{12} = 8\frac{12}{12} + \frac{3}{12} = 8\frac{15}{12} \\ -5\frac{5}{6} = 5\frac{10}{12} = \qquad -5\frac{10}{12} \\ \hline 3\frac{5}{12} \end{array}$$

More examples:

$$\begin{array}{r} 8\frac{1}{2} = 8\frac{2}{4} = 7\frac{4}{4} + \frac{2}{4} = 7\frac{6}{4} \\ -4\frac{3}{4} = 4\frac{3}{4} = \qquad -4\frac{3}{4} \\ \hline 3\frac{3}{4} \end{array}$$

$$\begin{array}{r} 10\frac{1}{5} = 10\frac{4}{20} = 9\frac{20}{20} + \frac{4}{20} = 9\frac{24}{20} \\ -6\frac{3}{4} = 6\frac{15}{20} = \qquad -6\frac{15}{20} \\ \hline 3\frac{9}{20} \end{array}$$

Help Pages

Solved Examples

Fractions (continued)

To multiply fractions, simply multiply the numerators together to get the numerator of the product. Then multiply the denominators together to get the denominator of the product. Make sure the answer is in simplest form.

Examples: Multiply $\frac{3}{5}$ by $\frac{2}{3}$.

$$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15} = \frac{2}{5}$$

1. Multiply the numerators.
2. Multiply the denominators.
3. Simplify the answer.

Multiply $\frac{1}{4}$ by $\frac{4}{5}$.

$$\frac{5}{8} \times \frac{4}{5} = \frac{20}{40} = \frac{1}{2}$$

Sometimes cancelling can be used when multiplying fractions. Look at the examples again.

$$\frac{\overset{1}{\cancel{3}}}{5} \times \frac{2}{\underset{1}{\cancel{3}}} = \frac{2}{5}$$

The 3's have a common factor — 3. Divide both of them by 3. Since, 3 ÷ 3 = 1, we cross out the 3's and write 1's in their place.

Now, multiply the fractions. In the numerator, 1 × 2 = 2 . In the denominator, 5 × 1 = 5.

The answer is $\frac{2}{5}$.

1. Are there any numbers in the numerator and the denominator that have common factors?
2. If so, cross out the numbers, divide both by that factor, and write the quotient.
3. Then, multiply the fractions as described above, using the quotients instead of the original numbers.

$$\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{5}}} = \frac{1}{2}$$

As in the other example, the 5's can be cancelled.

But here, the 4 and the 8 also have a common factor — 4.

$8 \div 4 = 2$ and $4 \div 4 = 1$.

After cancelling both of these, multiply the fractions.

The answer is $\frac{1}{2}$.

REMEMBER: Cancelling can be done up and down or diagonally, but NEVER sideways!

When **multiplying mixed numbers**, first change them into improper fractions.

Examples: Multiply $2\frac{1}{4}$ by $3\frac{1}{9}$.

$$2\frac{1}{4} \times 3\frac{1}{9} =$$

$$\frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{7}{\cancel{28}}}{\underset{1}{\cancel{9}}} = \frac{7}{1} = 7$$

1. Change each mixed number to an improper fraction.
2. Cancel wherever possible.
3. Multiply the fractions.
4. Put the answer in simplest form.

Multiply $3\frac{1}{8}$ by 4.

$$3\frac{1}{8} \times 4 =$$

$$\frac{25}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{1} = \frac{25}{2} = 12\frac{1}{2}$$

To **divide fractions**, take the reciprocal of the 2nd fraction, and then multiply that reciprocal by the 1st fraction. Simplify the answer.

Examples: Divide $\frac{1}{2}$ by $\frac{7}{12}$.

$$\frac{1}{2} \div \frac{7}{12} =$$

$$\frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{6}{\cancel{12}}}{7} = \frac{6}{7}$$

1. Keep the 1st fraction as it is.
2. Write the reciprocal of the 2nd fraction.
3. Change the sign to multiplication.
4. Cancel if possible and multiply.
5. Simplify the answer.

Divide $\frac{7}{8}$ by $\frac{3}{4}$.

$$\frac{7}{8} \div \frac{3}{4} =$$

$$\frac{7}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{3} = \frac{7}{6} = 1\frac{1}{6}$$

Help Pages

Solved Examples

Fractions (continued)

When **dividing mixed numbers**, first change them into improper fractions.

Example: Divide $1\frac{1}{4}$ by $3\frac{1}{2}$.

$$1\frac{1}{4} \div 3\frac{1}{2} =$$
$$\frac{5}{4} \div \frac{7}{2} =$$
$$\frac{5}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{7} = \frac{5}{14}$$

1. Change each mixed number to an improper fraction.
2. Keep the 1st fraction as it is.
3. Write the reciprocal of the 2nd fraction.
4. Change the sign to multiplication.
5. Cancel if possible and multiply.
6. Simplify the answer.

Decimals

When **comparing decimals**, look at two or more decimal numbers and decide which has the smaller or larger value. Sometimes, compare decimals by placing them in order from least to greatest or from greatest to least. Another way to compare is to use the symbols for "less than" (<), "greater than" (>), or "equal to" (=).

Example: Order these numbers from least to greatest. 0.561 0.506 0.165

1. Write the numbers in a column, lining up the decimal points.
2. Write zeros, if necessary, so all have the same number of digits.
3. Begin on the left and compare the digits.

0.561
0.506
0.165

Since they all have 3 digits, don't add zeros.

Beginning on the left, the fives are equal, but the one is less, so 0.165 is the smallest.

Then, look at the next digit. The zero is less than the six, so 0.506 is next smallest.

So, in order from least to greatest:

0.165, 0.506, 0.561

Example: Place these numbers in order from greatest to least. 0.44 0.463 0.045

0.440
0.463
0.045

After lining up the numbers, we must add a zero to 0.44 to make them all have the same number of digits.

Beginning on the left, the zero is smaller than the fours, so 0.045 is the smallest.

Look at the next digit. The four is smaller than the six, so 0.440 is the next smallest.

In order from greatest to least: 0.463, 0.440, 0.045

Help Pages

Solved Examples

Decimals (continued)

Rounding decimals is approximating. This means the decimal ends at a certain place value and is either rounded up (if it's closer to the next higher number) or kept the same (if it's closer to the next lower number). It might be helpful to look at the place-value chart for decimal numbers in these Help Pages.

Example: Round 0.574 to the tenths place.

There is a 5 in the rounding (tenths) place. → $0.\underline{5}74$

Since 7 is greater than 5, change the 5 to a 6. → $0.\underline{5}74$

Drop the digits to the right of the tenths place. → 0.6

1. Identify the number in the rounding place.
2. Look at the digit to its right.
3. If the digit is 5 or greater, increase the number in the rounding place by 1. If the digit is less than 5, keep the number in the rounding place the same.
4. Drop all digits to the right of the rounding place.

Example: Round 2.783 to the nearest hundredth.

$2.7\underline{8}3$ ← There is an 8 in the rounding place.

$2.7\underline{8}3$ ← Since 3 is less than 5, keep the rounding place the same

2.78 ← Drop the digits to the right of the hundredths place.

Adding and subtracting decimals is very similar to adding or subtracting whole numbers. The main difference is that the decimal points in the numbers must be lined up before beginning.

Examples: Find the sum of 3.14 and 1.2.

$$\begin{array}{r} 3.14 \\ +\ 1.20 \\ \hline 4.34 \end{array}$$

Add 55.1, 6.472, and 18.33.

$$\begin{array}{r} 55.100 \\ 6.472 \\ +18.330 \\ \hline 79.902 \end{array}$$

1. Line up the decimal points. Add zeros as needed.
2. Add (or subtract) the decimals.
3. Add (or subtract) the whole numbers.
4. Bring the decimal point straight down.

Examples: Subtract 3.7 from 9.3.

$$\begin{array}{r} 9.3 \\ -\ 3.7 \\ \hline 5.6 \end{array}$$

Find the difference of 4.1 and 2.88.

$$\begin{array}{r} 4.10 \\ -2.88 \\ \hline 1.22 \end{array}$$

Help Pages

Solved Examples

Decimals (continued)

When **multiplying a decimal by a whole number**, the process is similar to multiplying whole numbers.

Examples: Multiply 3.42 by 4.

```
   3.42 → 2 decimal places
 ×    4 → 0 decimal places
  13.68 → Place decimal point so there are (2 + 0) 2 decimal places.
```

Find the product of 2.3 and 2.

```
  2.3 → 1 decimal place
 ×  2 → 0 decimal places
  4.6 → Place decimal point so there is (1 + 0) 1 decimal place.
```

1. Line up the numbers on the right.
2. Multiply. Ignore the decimal point.
3. Place the decimal point in the product. (The total number of decimal places in the product must equal the total number of decimal places in the factors.)

The process for multiplying two decimal numbers is a lot like what was done above.

Examples: Multiply 0.4 by 0.6.

```
   0.4 → 1 decimal place
 × 0.6 → 1 decimal place
  0.24 → Place decimal point so there are (1 + 1) 2 decimal places.
```

Find the product of 2.67 and 0.3.

```
   2.67 → 2 decimal places
 ×  0.3 → 1 decimal places
  0.801 → Place decimal point so there are (2 + 1) 3 decimal places.
```

Sometimes it is necessary to add zeros in the product as placeholders in order to have the correct number of decimal places.

Example: Multiply 0.03 by 0.4.

```
   0.03 → 2 decimal places
 ×  0.4 → 1 decimal place
  0.012 → Place decimal point so there are (2 + 1) 3 decimal places.
```

Add a zero in front of the 12 so that there are 3 decimal places in the product.

The process for **dividing a decimal number by a whole number** is similar to dividing whole numbers.

Examples: Divide 6.4 by 8.

```
    0.8
 8)6.4
  -6 4
     0
```

Find the quotient of 20.7 and 3.

```
     6.9
 3)20.7
  -18
     27
    -27
      0
```

1. Set up the problem for long division.
2. Place the decimal point in the quotient directly above the decimal point in the dividend.
3. Divide. Add zeros as placeholders if necessary. (See examples below.)

Examples: Divide 4.5 by 6.

```
   0.75
 6)4.50   ← Add zero(es)
  -4 2↓
     30   ← Bring zero down.
   - 3 0
       0
```

Find the quotient of 3.5 and 4.

```
    0.875
 4)3.500   ← Add zero(es)
  -3 2↓↓
     30↓   ← Bring zero down. Keep dividing.
    -28↓
      20
    - 20
       0
```

Add zero(es)

Bring zero down.

Keep dividing.

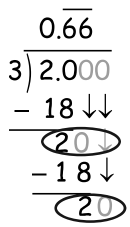
Help Pages

Solved Examples

Decimals (continued)

When dividing decimals the remainder is not always zero. Sometimes, the division continues on and on and the remainder begins to repeat itself. When this happens the quotient is called a **repeating decimal**.

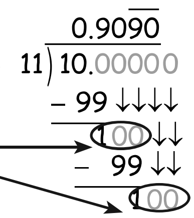
Examples: Divide 2 by 3. Divide 10 by 11.



Add zeros as needed.

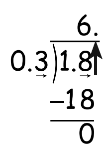
This pattern (with the same remainder) begins to repeat itself.

To write the final answer, put a bar in the quotient over the digits that repeat.



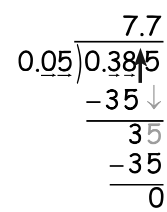
The process for **dividing a decimal number by a decimal number** is similar to other long division. The main difference is that the decimal point must move in both the dividend and the divisor the <u>same number of places</u> to the right.

Example: Divide 1.8 by 0.3.



1. Change the divisor to a whole number by moving the decimal point as many places to the right as needed.
2. Move the decimal in the dividend the same number of places to the right as in the divisor.
3. Put the decimal point in the quotient directly above the decimal point in the dividend.
4. Divide.

Divide 0.385 by 0.05.

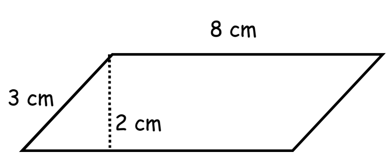


Geometry

Finding the **area of a parallelogram** is similar to finding the area of any other quadrilateral. The area of the figure is equal to the length of its base multiplied by the height of the figure.

Area of parallelogram = base × height or A = b × h

Example: Find the area of the parallelogram below.



1. Find the length of the base. (8 cm)
2. Find the height. (It is 2 cm. The height is always straight up and down – never slanted.)
3. Multiply to find the area. (16 cm^2)

So, A = 8 cm × 2 cm = 16 cm^2.

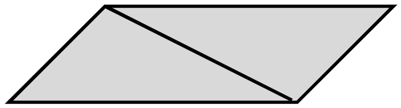
Help Pages

Solved Examples

Geometry (continued)

To find the **area of a triangle**, it is helpful to recognize that any triangle is exactly half of a parallelogram.

The whole figure is a parallelogram.

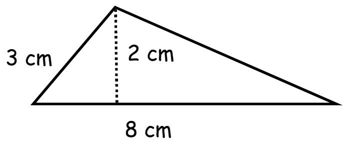


Half of the whole figure is a triangle.

So, the triangle's area is equal to half of the product of the base and the height.

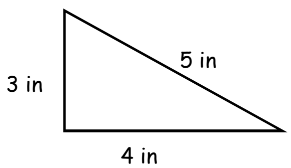
Area of triangle = $\frac{1}{2}$(base $\times$ height) or $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

Examples: Find the area of the triangles below.



1. Find the length of the base. (8 cm)
2. Find the height. (It is 2 cm. The height is always straight up and down, never slanted.)
3. Multiply them together and divide by 2 to find the area. (8 cm^2)

So, $A = \frac{8\text{ cm} \times 2\text{ cm}}{2} = 8\text{ cm}^2$.



The base of this triangle is 4 inches long. Its height is 3 inches. (Remember the height is always straight up and down!)

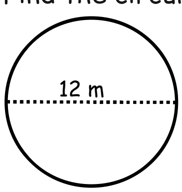
So, $A = 4\text{ in} \times 3\text{ in} \times \frac{1}{2} = 6\text{ in}^2$.

The **circumference of a circle** is the distance around the outside of the circle. Before finding the circumference of a circle, either its radius or its diameter must be known as well as the value of the constant, pi ($\neq$). $\pi = 3.14$ (rounded to the nearest hundredth)

With this information, the circumference can be found by multiplying the diameter by pi.

Circumference = $\pi \times$ diameter or $C = \pi d$

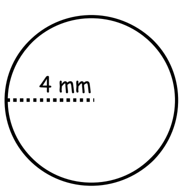
Examples: Find the circumference of the circles below.



1. Find the length of the diameter. (12 m)
2. Multiply the diameter by π. (12 m $\times$ 3.14)

So, $C = 12\text{ m} \times 3.14 = 37.68$ m.

Sometimes the radius of a circle is given instead of the diameter. Remember, the radius of any circle is exactly half of the diameter. If a circle has a radius of 3 feet, its diameter is 6 feet.



Since the radius is 4 mm, the diameter must be 8 mm.

Multiply the diameter by π. (8 mm $\times$ 3.14)

The product is the circumference. (25.12 mm)

So, $C = 8\text{ mm} \times 3.14 = 25.12$ mm.

Help Pages

Solved Examples

Ratio and Proportion

A ratio is used to compare two numbers. There are three ways to write a ratio comparing 5 and 7:

1. Word form ➡ 5 to 7
2. Fraction form ➡ $\frac{5}{7}$
3. Ratio form ➡ 5 : 7

Make sure that all ratios are written in simplest form. (Just like fractions!)

A proportion is a statement showing that two ratios are equal to each other. There are two ways to solve a proportion when a number is missing.

1. One way to solve a proportion is already familiar. Use the equivalent fraction method.

$$\frac{5}{8} = \frac{n}{64}$$

$n = 40$

So, $\frac{5}{8} = \frac{40}{64}$

To use Cross-Products:

1. Multiply downward on each diagonal.
2. Make the product of each diagonal equal to each other.
3. Solve for the missing variable.

2. Another way to solve a proportion is by using cross-products.

$$\frac{14}{20} = \frac{21}{n}$$

$$20 \times 21 = 14 \times n$$

$$420 = 14n$$

$$\frac{420}{14} = \frac{14n}{14}$$

$$30 = n$$

So, $\frac{14}{20} = \frac{21}{30}$

Who Knows?

Degrees in a right angle? ....(90)
A straight angle? ....(180)
Angle greater than 90°? .... (obtuse)
Less than 90°? ....(acute)
Sides in a quadrilateral? ....(4)
Sides in an octagon?............................ (8)
Sides in a hexagon? ....(6)
Sides in a pentagon? .... (5)
Sides in a heptagon? ....(7)
Sides in a nonagon? .... (9)
Sides in a decagon?.......................... (10)
Inches in a yard? ....(36)
Yards in a mile? ....(1,760)
Feet in a mile? ....(5,280)
Centimeters in a meter? ....(100)
Teaspoons in a tablespoon? .... (3)
Ounces in a pound? ....(16)
Pounds in a ton?............................(2,000)
Cups in a pint? .... (2)
Pints in a quart? .... (2)
Quarts in a gallon? ....(4)
Millimeters in a meter? .... (1,000)
Years in a century? ....(100)
Years in a decade? ....(10)
Celsius freezing? ....(0°C)
Celsius boiling? ....(100°C)
Fahrenheit freezing? ....(32°F)
Fahrenheit boiling? ....(212°F)
Number with only 2 factors? .... (prime)

Perimeter? ....(add the sides)
Area? ....(length x width)
Volume? ....(length x width x height)
Area of parallelogram?... (base x height)
Area of triangle? ....($\frac{1}{2}$ base x height)
Area of trapezoid ....($\frac{\text{base} + \text{base}}{2} \times \text{height}$)
Area of a circle? .... (πr2)
Circumference of a circle? ....(dπ)
Triangle with no sides equal? ....(scalene)
Triangle with 3 sides equal?...(equilateral)
Triangle with 2 sides equal? ....(isosceles)
Distance across the middle of a circle? ....(diameter)
Half of the diameter? .... (radius)
Figures with the same size and shape? ....(congruent)
Figures with same shape, different sizes? ....(similar)
Number occurring most often? ....(mode)
Middle number? ....(median)
Answer in addition? ....(sum)
Answer in division? ....(quotient)
Answer in subtraction? ......(difference)
Answer in multiplication? ....(product)