Summer Solutions.

Minutes a Day–Mastery for a Lifetime!

Standards-Based Mathematics 8

Help Pages

Help Pages

Vocabulary

absolute value	the distance between a number and zero on a number line. Example: the absolute value of negative seven is 7; it is written as $\|-7\|$. Absolute value is never negative.
additive inverse property	states that the sum of any number and its inverse is zero. $-5 + 5 = 0$ is an example of the additive inverse property.
adjacent angles	angles that share a common vertex and a common side, but do not overlap. Adjacent angles are created when two lines intersect; they are directly next to one another.
algebraic expression	a mathematical phrase written in numbers and symbols. Example: $2x + 5$.
alternate exterior angles	angles that lie on opposite sides of the transversal and outside two lines
alternate interior angles	angles that lie on opposite sides of the transversal and inside two lines
axis / axes	the lines that form the framework for a graph. The horizontal axis is called the *x*-axis; the vertical axis is called the *y*-axis.
cluster	is a collection of points that are close together. On a scatter plot all of the ordered pairs are gathered around a particular value.
coefficient	a number in front of a variable in an algebraic term. Example: $5x$ (5 is the coefficient.)
complementary angles	two angles whose sum is 90°
complex fraction	fractions that have fractions in the numerator and/or fractions in the denominator
congruent	figures with the same shape and the same size
constant	in an algebraic expression, a number that is not attached to a variable; a term that always has the same value. Example: In the expression $3x + 4$, the number 4 is a constant.
coordinates	an ordered pair of numbers that give the location of a point on a coordinate grid
coordinate plane/grid	a grid in which the location is described by its distances from two intersecting, straight lines called axes
correlation	a relationship between two things. Scatter plots can have positive, negative, or no correlation. *See* **negative association**, **no association**, and **positive association**.
corresponding angles	angles that are in the same position on two lines in relation to a transversal
cube root ($\sqrt[3]{\ }$)	one of three equal factors of a number. If $x^3 = y$, then x is the cube root of y. Example: numbers such as 8, 27, and 64 are perfect cubes because they are the cubes of integers.
dependent variable	a variable that is affected by the independent variable. In $y = 3x$, the value of y depends on the value of x. The dependent variable in this example is y.
dilation	a transformation in which a shape is scaled up or down
evaluate	to find the value of an expression
exponent	tells the number of times that a base is multiplied by itself. An exponent is written to the upper right of the base. Example: $5^3 = 5 \times 5 \times 5$. The exponent is 3.
exponential notation	an expression with an exponent. 4^3 is an example of exponential notation.
exterior angle	four outer angles formed by two lines cut by a transversal
face	a flat surface of a solid figure
function	a rule that pairs each number in a given set (the domain) with just one number in another set (the range). Example: the function $y = x + 3$ pairs every number with another number that is larger by 3. A function relates each input value, with exactly one output value.
hypotenuse	the side of the triangle that is opposite the right angle in a right triangle. When using the Pythagorean Theorem, the hypotenuse is *c*.
inequality	a statement that one quantity is different than another

Help Pages

Vocabulary

independent variable	a variable that affects the dependent variable. For example, the independent variable is x in $y = 3x$. When x is 2, $y = 6$; when x is 0.5, $y = 1.5$.
input value	in a function, the x-value or the domain
integers	the set of whole numbers, positive, negative, and zero. A set of integers includes zero, the counting numbers, and their opposites.
interest	amount paid or earned for the use of money over time
interior angle	inside angle of a polygon
irrational number	a number that cannot be written as the ratio of two whole numbers. The decimal form of an irrational number is neither terminating nor repeating. Examples: $\sqrt{2}$ and π
legs of a triangle	the sides of a right triangle that are adjacent to the right angle. When using the Pythagorean Theorem the legs are a and b.
like terms	terms that have the same variable and are raised to the same power. Like terms can be combined (added or subtracted), whereas unlike terms cannot.
linear association	a pattern of association where the data points on a scatter plot lie close to a line
linear expression	an algebraic expression in which a single variable is raised to the first power. It is sometimes simply called an expression. $2x$ and $6x + 4$ are examples of linear expressions.
line of best fit	a line that is very close to most of the data in a scatter plot. About half of the ordered pairs are above and half are below the line.
negative association	also known as negative correlation. A pattern of association in which ordered pairs on a scatter plot move downward in a negative pattern. As x increases, y decreases.
negative numbers	all the numbers less than zero. (Zero is neither positive nor negative.) A negative number has a negative sign (–) in front of it.
no association	also known as no correlation, in which ordered pairs on a scatter plot do not form a pattern.
nonlinear association	a pattern of association where the data points on a scatter plot do not lie close to a line; the shape is more of a curve
order of operations	a rule that tells the order in which to perform operations in an equation
origin	the point where the x-axis and y-axis intersect; the point (0, 0)
outlier	an ordered pair that lies outside the cluster of ordered pairs
output value	in a function, the y-value or range
patterns of association	a pattern between two variables. Patterns of association can be positive, negative, or have no association. Other ways to describe a pattern of association would be linear or nonlinear.
percent	the ratio of any number to 100; the symbol for percent is %
perfect square	the result of squaring a whole number. Examples: $2^2 = \mathbf{4}$, $6^2 = \mathbf{36}$, $9^2 = \mathbf{81}$
pi (π)	the ratio of circumference to diameter of a circle. Pi is approximately 3.14, or $\frac{22}{7}$.
positive association	also known as positive correlation. A pattern of association in which ordered pairs on a scatter plot move upward in a positive pattern. As x increases, y increases.
positive numbers	all numbers greater than 0; sometimes a (+) is written in front of a positive number
principal	amount of money borrowed or invested
prism	a three-dimensional figure that has two identical, parallel bases, and three or more rectangular faces
proportion	a statement that two ratios (or fractions) are equal

Help Pages

Vocabulary

pyramid	a three-dimensional figure with one base that is a polygon (rectangle, triangle, pentagon, hexagon, etc.) and whose faces are all triangles
Pythagorean Theorem	a way to find the length of any side of a right triangle. The square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b. The formula for the Pythagorean Theorem is $a^2 + b^2 = c^2$.
radical sign	the symbol used to indicate a positive square root $\sqrt{}$
radius	the distance from any point on the circle to the center. The radius is half of the diameter.
rational number	a number that can be written as the ratio of two whole numbers. Example: 7 is a rational number; it can be written as $\frac{7}{1}$; 0.25 is rational; it can be written as $\frac{1}{4}$.
reflection	a transformation where a figure is flipped over a line. It is also known as flipped or mirrored.
relative frequency	the ratio of the number of experimental successes to the total number of experimental attempts $\frac{\text{the number of times a condition is observed}}{\text{total number of observations}}$
rise	the vertical change between any two points on a line
rotation	a transformation in which a figure is turned about a fixed point
run	the horizontal change between any two points on a line
sample space	the set of possible outcomes of a probability event
scale	the relationship between two lengths
scale factor	the ratio of the corresponding side lengths of two similar shapes
scatter plot	a graph that shows the relationship between two sets of data variables graphed as ordered pairs on a coordinate plane
scientific notation	a way to express very large or very small numbers. The product of a number x, where $1 \leq x < 10$, and a power of ten. Example: 1.23×10^3
similar	polygons that can be formed from the other through a set of transformations or dilations. For polygons to be similar they have to have the same shape, meaning their corresponding angles are congruent and the measures of their corresponding sides are proportional.
slope	the rate of change between any two points on a line. The ratio of the rise (vertical change) to the run (horizontal change).
square root ($\sqrt{}$)	one of two equal factors of a number. If $x^2 = y$, then x is the square root of y.
supplementary angles	two angles whose sum is 180°
surface area	the sum of the areas of the faces of a solid figure
term	a part of an algebraic expression; a number, variable, or combination of the two; terms are separated by signs, such as +, =, or –
translation	a transformation that slides or moves a figure from one location to another without turning
transversal	any line that passes through two lines
undefined	when 0 is in the denominator of a fraction. For example: $\frac{6}{0}$ or $6 \div 0 \neq 0$ because $0 \times 0 \neq 6$
unit rate	a ratio of two values; in a unit rate, the number in the denominator is 1
variable	an unknown or a symbol that stands for an unknown value; a variable can change. In an algebraic expression one must define a variable. This means to choose a letter or symbol to stand for an unknown value.
volume	the number of cubic units it takes to fill a solid; volume is expressed in cubic units
***y*-intercept**	the y-coordinate of a line where the line crosses the y-axis

Help Pages

Expressions and Equations 8.EE.1 – 8.EE.8

Solving Algebraic Equations

An **equation consists of two expressions separated by an equal sign.** You have worked with simple equations for a long time: 2 + 3 = 5. More complicated equations involve variables that replace a number. To solve an equation like this, you must figure out which number the variable stands for. A simple example is when $2 + x = 5$, $x = 3$. Here, the variable, *x*, stands for 3. Sometimes an equation is not so simple. No matter how complicated the equation, <u>the goal is to work with the equation until all the numbers are on one side and the variable is alone on the other side</u>. To check your answer, put the value of *x* back into the original equation.

Solving an **equation with a variable on one side:**

1. Look at the side of the equation that has the variable. If there is a number added to or subtracted from the variable, it must be removed. In the first example, 13 is added to *x*.
2. To remove 13, add its opposite (–13) to both sides of the equation.
3. Add downward. *x* plus "nothing" is *x*. 13 plus –13 is zero. 27 plus –13 is 14.
4. Once the variable is alone on one side of the equation, the equation is solved. The bottom line tells the value of *x*. $x = 14$.

Example 1: Solve for *x*.

❶ → $x + 13 = 27$

$\underline{\;\;-13 = -13}$ ← ❷

❹ → $x \quad = 14$

Check: $14 + 13 = 27$ ✓

Example 2: Solve for *x*.

❶ → $a - 22 = -53$

$\underline{+22 = +22}$ ← ❷

❹ → $a \quad = -31$

Check: $-31 - 22 = -53$ ✓

In the next examples, a number is either multiplied or divided by the variable (not added or subtracted).

1. Look at the side of the equation that has the variable. If there is a number multiplied by or divided into the variable, it must be removed. In the first example, 3 is multiplied by *x*.
2. To remove 3, divide both sides by 3. You divide because it is the inverse operation from the one in the equation (multiplication).
3. Follow the rules for multiplying or dividing integers. 3*x* divided by 3 is *x*. 39 divided by 3 is 13.
4. Once the variable is alone on one side of the equation, the equation is solved. The bottom line tells the value of *x*. $x = 13$.

Example 1: Solve for *x*.

❶ → $3x = 39$

❷ → $\frac{3x}{3} = \frac{39}{3}$

❹ → $x = 13$

Check:
$3(13) = 39$
$39 = 39$ ✓

Example 2: Solve for *n*.

❶ → $\frac{n}{6} = -15$

❷ → $\frac{n}{6}(6) = -15(6)$

❹ → $n = -90$

Check:
$\frac{-90}{6} = -15$
$-15 = -15$ ✓

The next set of examples also have a variable on only one side of the equation. These, however, are a bit more complicated, because they will require two steps in order to get the variable alone.

1. Look at the side of the equation that has the variable. There is a number (2) multiplied by the variable, and there is a number added to it (5). Both of these must be removed. Always begin with addition/subtraction. To remove the 5 we must add its opposite (–5) to both sides.
2. To remove the 2, divide both sides by 2. You divide because it is the opposite operation from the one in the equation (multiplication).
3. Follow the rules for multiplying or dividing integers. 2*x* divided by 2 is *x*. 8 divided by 2 is four.
4. Once the variable is alone on one side of the equation, the equation is solved. The bottom line tells the value of *x*. $x = 4$.

Example 1: Solve for *x*.

$2x + 5 = 13$

❶ → $\underline{-5 = -5}$

$2x = 8$

$\frac{2x}{2} = \frac{8}{2}$ ← ❷

❹ → $x = 4$

Check:
$2(4) + 5 = 13$
$8 + 5 = 13$
$13 = 13$ ✓

Example 2: Solve for *n*.

$\frac{n}{3} - 7 = 5$

$+7 = +7$

$(3)\frac{n}{3} = 12(3)$

$n = 36$

Check:
$\frac{36}{3} - 7 = 5$
$12 - 7 = 5$
$5 = 5$ ✓

Help Pages

Expressions and Equations 8.EE.1 – 8.EE.8

Solving Algebraic Equations (continued)

This multi-step equation also has a variable on only one side. To get the variable alone, though, requires several steps.

Example: Solve for x. $3(2x+3)=21$

Method 1

1. Use the distributive property.
2. Use the directions for the third set of examples on the previous page to complete the problem.

$$\begin{array}{rcl}
3(2x+3) & = & 21 \\
6x+9 & = & 21 \\
\underline{-9} & \underline{=} & \underline{-9} \\
6x & = & 12 \\
\frac{6x}{6} & = & \frac{12}{6} \\
x & = & 2
\end{array}$$

Method 2

1. Use division to isolate the factor $(2x+3)$.
2. Use the directions for the third set of examples on the previous page to complete the problem.

$$\begin{array}{rcl}
3(2x+3) & = & 21 \\
\frac{3(2x+3)}{3} & = & \frac{21}{3} \\
2x+3 & = & 7 \\
\underline{-3} & \underline{=} & \underline{-3} \\
2x & = & 4 \\
x & = & 2
\end{array}$$

$$\begin{array}{rrcl}
\text{Check} & 3[2(2)+3] & = & 21 \\
 & 3(4+3) & = & 21 \\
 & 3(7) & = & 21 \\
 & 21 & = & 21
\end{array}$$

When **solving an equation with a variable on both sides**, the goals are the same: to get the numbers on one side of the equation and to get the variable alone on the other side.

Example: Solve for x. $2x+4=6x-4$

1. Since there are variables on both sides, the first step is to remove the "variable term" from one of the sides. To remove $2x$ from the left side, add $-2x$ to both sides.
2. Next, remove the number added to the variable side by adding its opposite. To remove -4, add $+4$ to both sides.
3. The variable still has a number multiplied by it (4), which can be removed by dividing both sides by 4.

$$\begin{array}{rcll}
2x+4 & = & 6x-4 & \\
\underline{-2x} & \underline{=} & \underline{-2x} & \longleftarrow \text{❶} \\
4 & = & 4x-4 & \\
\underline{+4} & \underline{=} & \underline{+4} & \longleftarrow \text{❷} \\
8 & = & 4x & \\
\frac{8}{4} & = & \frac{4x}{4} & \longleftarrow \text{❸} \\
2 & = & x &
\end{array}$$

Solving Linear Equations — Number of Solutions

Linear equations can have 0, 1, or infinite solutions.

- If you solve for x and get one answer, then the equation has **one solution**.
- If you solve for x and find that 0 = 0 (in other words, everything has cancelled out), then the equation has **infinite solutions**.
- A linear equation has **no solutions** if you solve for x and you are left with an equation that tells you a = b when a and b are different numbers.

Example: Solve $x+21=24$. How many solutions does this equation have?

$$\begin{array}{rcl}
x+21 & = & 24 \\
\underline{-21} & \underline{=} & \underline{-21} \\
x & = & 3
\end{array}$$

This equation has one solution.

Example: Solve $2(10-x)=-2x+20$. How many solutions does this equation have?

$$\begin{array}{rcl}
2(10-x) & = & -2x+20 \\
20-2x & = & -2x+20 \\
\underline{-20} & \underline{=} & \underline{-20} \\
-2x & = & -2x \\
\underline{+2x} & \underline{=} & \underline{+2x} \\
0 & = & 0
\end{array}$$

0 = 0; This equation has infinite solutions.

Example: Solve $x+7=x+20$. How many solutions does this equation have?

$$\begin{array}{rcl}
x+7 & = & x+20 \\
\underline{-7} & \underline{=} & \underline{-7} \\
x & = & x+13 \\
\underline{-x} & \underline{=} & \underline{-x} \\
0 & = & 13
\end{array}$$

According to the equation $0=13$; $0 \neq 13$, therefore this equation has no solutions.

Help Pages

Expressions and Equations 8.EE.1 – 8.EE.8

Order of Operations

When evaluating a numerical expression containing multiple operations, use a set of rules called the order of operations. The **order of operations** determines the order in which operations should be performed.

The order of operations is as follows:

Step ❶: Grouping Symbols
Step ❷: Exponents
Step ❸: Multiplication/Division (left to right in the order that they occur)
Step ❹: Addition/Subtraction (left to right in the order that they occur)

If grouping symbols are enclosed within other grouping symbols, work from the inside out.

To remember the order, use the mnemonic device **"Please Excuse My Dear Aunt Sally."**

Use the following examples to help you understand how to use the order of operations.

Example: $2^2 + 6 \times 5$

To evaluate this expression, work through the steps using the order of operations.

Since there are no parentheses, skip Step 1.

According to Step 2, do exponents next.

Step 3 is multiplication/division.

Next, Step 4 says to do addition/subtraction.

$2^2 + 6 \times 5$ ← Step ❷ Exponents
$4 + 6 \times 5$ ← Step ❸ Multiplication/Division
$4 + 30$ ← Step ❹ Addition/Subtraction
34

Example: $42 \div 6 - 3 + 4 - 16 \div 2$

Do multiplication/division first (in the order they occur).
Do addition/subtraction next (in the order they occur).

$42 \div 6 - 3 + 4 - 16 \div 2$ ← Step ❸ Multiply/Divide
$7 - 3 + 4 - 16 \div 2$
$7 - 3 + 4 - 8$
$4 + 4 - 8$ ← Step ❹ Add/Subtract
$8 - 8$
0

Example: $5(2 + 4) + 15 \div (9 - 6)$

Do operations inside of parentheses first.
Do multiplication/division first (in the order they occur).
Do addition/subtraction next (in the order they occur).

$5(2+4)+15\div(9-6)$ ← Step ❶ Parentheses (Do operations inside first.)
$5(6)+15\div(3)$ ← Step ❸ Multiply/Divide (in the order they occur).
$30+5$ ← Step ❹ Add
35

Example: $4[3 + 2(7 + 5) - 7]$

Brackets are treated as parentheses. Start from the innermost parentheses first.
Then work inside the brackets.

$4[3+2(7+5)-7]$ ← Step ❶ Parentheses (including brackets) Solve innermost parentheses first. Then work inside the brackets.
$4[3+2(12)-7]$
$4[3+24-7]$
$4[27-7]$
$4[20]$
80

Example: Place grouping symbols to make this equation true. $36 \div 4 + 5 = 4$

	Without Grouping Symbols	With Grouping Symbols	
Without parentheses the first step is 36÷4. →	$36 \div 4 + 5 = ?$	$36 \div (4+5) = ?$	← With parentheses the first step is 4 + 5.
	$9 + 5 = ?$	$36 \div 9 = ?$	
	$9 + 5 = 14$	$36 \div 9 = 4$	

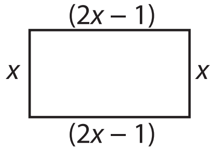
Help Pages

Expressions and Equations　　8.EE.1 – 8.EE.8

Finding the Perimeter of a Rectangle

A rectangle has four right angles and 2 pairs of parallel sides. The distance around the outside of a rectangle is the **perimeter**. To find the perimeter of a rectangle, add the lengths of the sides and combine like terms.

Example:



$2x + 2(2x - 1) = 2x + 4x - 2$

$6x - 2$

$6x - 2$ (If $x = 6$, then P = 34)

Inequalities

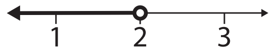
An inequality is a statement that one quantity is different than another (usually larger or smaller). The symbols showing inequality are <, >, ≤, and ≥ (less than, greater than, less than or equal to, and greater than or equal to.) An inequality is formed by placing one of the inequality symbols between two expressions. The solution of an inequality is the set of numbers that can be substituted for the variable to make the statement true.

A simple inequality is $x \le 4$. The solution set, {..., 2, 3, 4}, includes all numbers that are either less than four or equal to four.

Inequalities can be graphed on a number line. For < and >, use an open circle; for ≤ and ≥, use a closed circle.

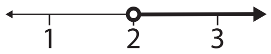
Example 1

$x < 2$



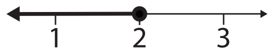
Example 2

$x > 2$



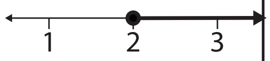
Example 3

$x \le 2$



Example 4

$x \ge 2$



Some inequalities are solved using only addition or subtraction. The approach to solving them is similar to that used when solving equations. The goal is to get the variable alone on one side of the inequality and the numbers on the other side.

Examples: Solve $x - 4 < 8$.

$$\begin{array}{rcr} x - 4 & < & 8 \\ +4 & & +4 \\ \hline x & < & 12 \end{array}$$

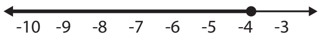
1. To get the variable alone, add the opposite of the number that is with it to both sides.
2. Simplify both sides of the inequality.
3. When dividing/multiplying an inequality by a negative number, the direction of the inequality sign changes.
4. Graph the solution on a number line. For < and >, use an open circle; for ≤ and ≥, use a closed circle.

Solve $-2y + 3 \ge 11$.

$$\begin{array}{rcr} -2y + 3 & \ge & 11 \\ -3 & & -3 \\ \hline -2y & \ge & 8 \end{array}$$

$$\frac{-2y}{-2} \le \frac{8}{-2}$$

$$y \le -4$$



Some inequalities are solved using only multiplication or division. The approach to solving them is also similar to that used when solving equations. Here, too, the goal is to get the variable alone on one side of the inequality and the numbers on the other side.

Example: Solve $8n < 56$.

$$\frac{8n}{8} < \frac{56}{8}$$

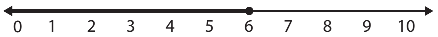
$$n < 7$$

1. Check to see if the variable is being multiplied or divided by a number.
2. Use the same number, but do the inverse operation on both sides.
3. Simplify both sides of the inequality.
4. Graph the solution on a number line. For < and >, use an open circle; for ≤ and ≥, use a closed circle.

Some inequalities must be solved using both addition/subtraction and multiplication/division. In these problems, the addition/subtraction is always done first.

Example: Solve $2x - 6 \le 6$.

$$\begin{array}{rcr} 2x - 6 & \le & 6 \\ +6 & & +6 \\ \hline 2x & \le & 12 \end{array} \qquad \begin{array}{c} \dfrac{2x}{2} \le \dfrac{12}{2} \\ x \le 6 \end{array}$$



Help Pages

Expressions and Equations 8.EE.1 – 8.EE.8

Exponents

An exponent is a small number to the upper right of another number (the **base**). Exponents are used to show that the base is a repeated factor.

Example: 2^4

2^4 is read "two to the fourth power."

The base (2) is a factor many times.

The exponent (4) tells how many times the base is a factor.

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

Example: 9^3

9^3 is read "nine to the third power" and means $9 \times 9 \times 9 = 729$

Until now, exponents have been positive numbers. What if the **exponent is zero or a negative number**? The rules for dealing with negative or zero exponents are as follows:

- *a* to the zero power is 1. $a^0 = 1, a \neq 0,$ so $7^0 = 1$
- a^{-n} is the reciprocal of a^n. $a^{-n} = \frac{1}{a^n}; a \neq 0; 9^{-1}; \frac{1}{9}$
- a^n is the reciprocal of a^{-n}. $a^n = \frac{1}{a^{-n}}; a^{-n}; a \neq 0; 4^1 = \frac{1}{4^{-1}}$

Examples: Evaluate the following: A) $(-10)^0$ B) $\left(\frac{1}{4}\right)^{-2}$ C) $\frac{1}{8^{-2}}$

A) $(-10)^0 = 1$ B) $\left(\frac{1}{4}\right)^{-2} = \frac{1}{\left(\frac{1}{4}\right)^2} = \frac{1}{\left(\frac{1}{16}\right)} = 16$ C) $\frac{1}{8^{-2}} = 8^2 = 64$

When **multiplying exponential terms** that have the same base, keep the base and **add** the exponents.

Examples: $a^2 \times a^3 = a^{2+3} = a^5$ $x^4 \times x^5 = x^{4+5} = x^9$

When **dividing exponential terms** with the same base, keep the base and **subtract** the exponents.

Example: $\frac{a^6}{a^2} = \frac{a \times a \times a \times a \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a}} = a \times a \times a \times a = a^4 = a^{6-2}$

Example: $\frac{b^{10}}{b^3} = b^{10-3} = b^7$

When **raising an exponential term to a power**, keep the base and **multiply** the exponents.

Examples: $\left(a^2\right)^3 = a^{2\times3} = a^6$ $\left(x^4\right)^4 = x^{4\times4} = x^{16}$

Sometimes the entire quotient is raised to a power. In that case, apply the exponent to both the numerator and denominator, simplify each of them, and then divide, if possible.

Example: $\left(\frac{2x^2}{5y}\right)^3 = \frac{\left(2x^2\right)^3}{\left(5y\right)^3} = \frac{2^3 \times \left(x^2\right)^3}{5^3 \times y^3} = \frac{8x^6}{125y^3}$

Example: $\left(\frac{3x^2}{9y^2}\right)^2 = \frac{\left(3x^2\right)^2}{\left(9y^2\right)^2} = \frac{3^2 \times \left(x^2\right)^2}{9^2 \times \left(y^2\right)^2} = \frac{\cancel{9}x^4}{\underset{9}{\cancel{81}}y^4} = \frac{x^4}{9y^4}$

Help Pages

Expressions and Equations 8.EE.1 – 8.EE.8

Square Roots

The **square root** of a number is one of the number's two equal factors. If $x^2 = y$, then x is the square root of y. Squaring a number and taking the square root of a number are inverse operations. The $\sqrt{\ }$ symbol is called the **radical sign**. When placed in front of a number, the radical sign means the square root of that number. $\sqrt{25}$ reads "The square root of 25 is 5." The result of squaring a whole number is called a **perfect square.** $2^2 = \mathbf{4}$, $6^2 = \mathbf{36}$, $9^2 = \mathbf{81}$ are examples of perfect squares.

A positive or negative symbol in front of the radical sign tells the sign of the answer:

Examples: $\pm\sqrt{4} = \pm 2$ $-\sqrt{9} = -3$ $\sqrt{16} = 4$ (Remember, all numbers with no sign in front of them are positive.)

Every number has both a positive and a negative square root.

Example: $\sqrt{25} = \pm 5$ (because $5 \times 5 = 25$ and $-5 \times -5 = 25$)

The **square root of a negative number** does not exist in the set of real numbers, because no number times itself equals a negative number.

Example: $-3 \times -3 = +9$ and $+3 \times +3 = +9$

Fractions and Square Roots

The square root of a fraction is the same as the square root of the numerator over the square root of the denominator.

Example: $\sqrt{\frac{4}{25}} = \pm\frac{\sqrt{4}}{\sqrt{25}} = \pm\frac{2}{5}$

Solving Equations with Square Roots

Squaring a number and taking the square root of a number are inverse operations. To **isolate a variable that is squared**, you would take the square root. Remember that what you do to one side of the equation, you must do to the other side.

Example: $x^2 = 144$. Solve for *x*.

1. To isolate the variable, find the square root of x^2.
2. What you do to one side of the equation, you must do to the other side.
3. To check, substitute the *x* value into the given equation.

$$x^2 = 144$$
$$\sqrt{x^2} = \pm\sqrt{144}$$
$$x = \pm 12$$

Check: $12^2 = 144$ ✓

Cube Roots

The **cube root** of a number is one of its three equal factors. If $x^3 = y$, then *x* is the cube root of *y*. The symbol $\sqrt[3]{\ }$ is used to indicate the cubed root of a number.

Numbers such as 8, 27, and 64 are **perfect cubes** because they are the cubes of integers.

Example: $8 = 2 \times 2 \times 2$ or 2^3, so the cube root of 8 is 2. What is the cube root of 27? 64?

$27 = \underline{\ 3\ } \times \underline{\ 3\ } \times \underline{\ 3\ } = 3^3$ $\sqrt[3]{27} = 3$ $64 = \underline{\ 4\ } \times \underline{\ 4\ } \times \underline{\ 4\ } = 4^3$ $\sqrt[3]{64} = 4$

Unlike a square root, a negative cube root <u>can</u> exist. Remember $\sqrt{-64}$ does not exist because $-8 \times -8 = +64$, but $\sqrt[3]{-64}$ (the cube root of negative 64) is possible because $-4 \times -4 \times -4 = -64$.

Help Pages

Expressions and Equations 8.EE.1 – 8.EE.8

Solving Equations with Cube Roots

Cubing a number and taking the cube root of a number are inverse operations. To **isolate a variable that is cubed**, you would take the cube root.

Example: $x^3 = 729$. Solve for x.

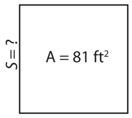
1. To isolate the variable, find the cube root of x^3.
2. What you do to one side of the equation, you must do to the other side.
3. To check, substitute the x value into the given equation.

$$x^3 = 729$$
$$\sqrt[3]{x^3} = \sqrt[3]{729}$$
$$x = 9$$
$$\text{Check: } 9^3 = 729 ✓$$

Applying Square and Cube Roots to Measurement

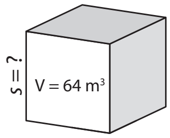
The square root of the **area of a square** is the length of any of its sides. The cube root of the **volume of a cube** is the length of any of its sides .

Example: The area of a square is 81 square feet. What is the length of one of its sides?



$$A = s^2 = 81 \text{ ft}^2$$
$$\sqrt{s^2} = \sqrt{81}$$
$$s = 9 \text{ ft}$$

Example: The volume of a cube is 64 cubic meters. What is the length of one of its sides?



$$V = s^3 = 64 \text{ m}^3$$
$$\sqrt[3]{s^3} = \sqrt[3]{64}$$
$$s = 4 \text{ m}$$

Scientific Notation

Scientific notation is a shorthand method for representing numbers that are either very large or very small — numbers that have many zeroes. Numbers like 5,000,000,000 and 0.000000023 have so many zeroes that it is not convenient to write them this way. Scientific notation removes the "placeholder" zeroes and represents them as powers of 10. Numbers in scientific notation always have the form $c \times 10^n$, where $1 \le c < 10$ and n is an integer.

Example: Convert 5,000,000,000 and 0.000000023 to scientific notation.

5,000,000,000 0.000000023

5×10^9 2.3×10^{-8}

1. First locate the decimal point. Remember, if the decimal point isn't shown, it is to the right of the last digit.
2. Move the decimal point (either left or right) until the number is at least 1 and less than 10.
3. Count the number of places you moved the decimal point. This is the exponent.
4. If you moved the decimal to the right, the exponent will be negative; if you moved it to the left, the exponent will be positive.
5. Write the number multiplied by a power of ten.

Scientific Notation and Calculators

When using a **scientific calculator**, it is very easy to work with scientific notation. Each calculator has a specific key that is used to enter scientific notation. (The key varies, depending on the brand, but they all work the same way!) A few examples are `EXP`, `×10^x`, `EE`. For this lesson, we'll use `EE`, but yours may be different depending on your calculator.

A number in scientific notation consists of a coefficient multiplied by 10 to a power. To enter this number into your calculator, type the coefficient (3.8), then press the `EE` key, and finally, enter the power of ten (5).

$$\underbrace{3.8}_{\text{coefficient}} \times 10^{\overbrace{5}^{\text{power of 10}}}$$

Notice how the calculator displays this number. When the calculator reads this way, you must recognize it as scientific notation, and write it as 3.8×10^5.

```
3.8E5
```

Keep in mind that either (or both) the coefficient or the power of ten can be negative. On a scientific calculator, the negative key is <u>not the same</u> as the minus key. The negative key usually looks like `(–)` or `(+/–)`.

Help Pages

Expressions and Equations 8.EE.1 – 8.EE.8

Scientific Notation and Calculators

Examples:

- Use your scientific calculator to enter 6.4×10^3. What is shown on the display? `6.4E3`

  Now press ENTER. What is shown on the display now? `6400`

- Use your scientific calculator to enter 4.23×10^{-4}. What is shown on the display? `4.23E-4`

  Now press ENTER. What is shown on the display now? `0.000423`

- Using your scientific calculator, multiply (4.2×10^5) by (8.3×10^{-9}). Write your answer in proper scientific notation.

1. Use the EE key to enter your numbers. Your screen should look like the example to the right.
2. Press ENTER. Note how the answer is displayed.
3. Display the answer in scientific notation by moving the decimal the appropriate number of places.

`4.2E5*8.3E-9`

`0.003486`

$0.\underbrace{003}_{1\,2\,3}486$

3.486×10^{-3}

Multiplication/Division

If you're using a scientific calculator, you simply enter the numbers as previously instructed. However, even if you don't have a scientific calculator available, multiplying or dividing with scientific notation is fairly straightforward.

Example: Solve. $(4.2 \times 10^3)(3.3 \times 10^2)$

1. Multiply the coefficients.
2. Multiply the powers of 10.
3. Put them together and express them in the proper form.

$$4.2 \times 3.3 = 13.86$$
$$10^3 \times 10^2 = 10^{3+2} = 10^5$$
$$13.86 \times 10^5 = 1.386 \times 10^6$$

Sometimes one of the numbers is not in scientific notation; it must be put in scientific notation before multiplying.

Example: $4{,}300{,}000 \times (6.4 \times 10^5)$

1. Convert all numbers to scientific notation.
2. Multiply the coefficients.
3. Multiply the powers of 10.
4. Put them together and express them in the proper form.

Step ❶ $4{,}300{,}000 = 4.3 \times 10^6$

$(4.3 \times 10^6) \times (6.4 \times 10^5)$

Step ❷ $4.3 \times 6.4 = 27.52$

Step ❸ $10^6 \times 10^5 = 10^{6+5} = 10^{11}$

Step ❹ $27.52 \times 10^{11} = 2.752 \times 10^{12}$

Example: $\dfrac{8.3 \times 10^6}{2 \times 10^3}$

1. Divide the coefficients.
2. Divide the powers of 10.
3. Put them together and express them in the proper form.

Step ❶ $8.3 \div 2 = 4.15$

Step ❷ $10^6 \div 10^3 = 10^{6-3} = 10^3$

Step ❸ 4.15×10^3

Example: $73{,}000{,}000 \div 3.1 \times 10^4$ Round to the nearest hundredth.

1. Convert all numbers to scientific notation.
2. Divide the coefficients.
3. Divide the powers of 10.
4. Put them together and express them in the proper form.

Step ❶ $73{,}000{,}000 = 7.3 \times 10^7$

$(7.3 \times 10^7) \div (3.1 \times 10^4) = ?$

Step ❷ $7.3 \div 3.1 \cong 2.35$

Step ❸ $10^7 \div 10^4 = 10^{7-4} = 10^3$

Step ❹ 2.35×10^3

Division with real world problems

The mass of a proton is 1.67×10^{-27} kg. If an electron's mass is 8.55×10^{-31} kg, about how many times heavier is a proton than an electron? Express your answer in scientific notation.

$$\frac{1.67 \times 10^{-27} \text{ (proton's mass)}}{8.55 \times 10^{-31} \text{ (electron's mass)}} = 0.1953 \times 10^4 = 1.953 \times 10^3 \text{ (or just under 2,000 times) heavier}$$

Help Pages

Expressions and Equations 8.EE.1 – 8.EE.8

Scientific Notation and Calculators (continued)

Addition/Subtraction

Whenever decimals are added or subtracted, the numbers must be aligned by place value. In scientific notation, we can think of the exponent as representing the place value. So, before numbers can be added or subtracted, the numbers must be adjusted so that the exponent is the same.

Example: Evaluate. Express the result in scientific notation. $(5.12 \times 10^3) + (3.34 \times 10^2)$

1. Adjust one of the numbers so that the exponents are the same.
2. Add the coefficients.
3. Keep the power of 10.
4. Finally, put them together and express them in the proper form.

Step ❶ $5.12 \times 10^3 = 51.2 \times 10^2$

$(51.2 \times 10^2) + (3.34 \times 10^2)$

Step ❷ $51.2 + 3.34 = 54.54$

Step ❸ Keep 10^2

Step ❹ $54.54 \times 10^2 = 5.454 \times 10^3$

Example: Solve. Express the result in scientific notation. Round to the nearest hundredth. $(9.24 \times 10^7) - (2.214 \times 10^6)$

1. Adjust one of the numbers so that the exponents are the same.
2. Subtract the coefficients.
3. Keep the power of 10.
4. Finally, put them together and express them in the proper form.

Step ❶ $2.214 \times 10^6 = 0.2214 \times 10^7$

$(9.24 \times 10^7) - (0.2214 \times 10^7)$

Step ❷ $9.24 - 0.2214 = 9.0186$

Step ❸ Keep 10^7

Step ❹ 9.0186×10^7

Sometimes one of the numbers is not in scientific notation. To solve these problems, the number must be put into scientific notation before adding or subtracting.

Example: Evaluate. Express the result in scientific notation. Round to the nearest hundredth. $523{,}000 + 3.4 \times 10^6$

1. Convert all numbers to scientific notation.
2. Adjust one of the numbers so that the exponents are the same.
3. Add the coefficients.
4. Keep the power of 10.
5. Finally, put them together and express them in the proper form.

Step ❶ $523{,}000 = 5.23 \times 10^5$

Step ❷ $= 0.523 \times 10^6$

$(0.523 \times 10^6) + (3.4 \times 10^6)$

Step ❸ $0.523 + 3.4 = 3.923$

Step ❹ Keep 10^6

Step ❺ 3.92×10^6

Graphing — Slope

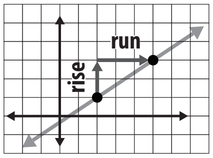
The slope of a line includes both a magnitude and a sign. The magnitude (size of the number) describes the incline, or steepness of the line. The sign (positive or negative) indicates the direction of the line. A line which moves upward from left to right will have a positive sign. A line which moves downward from left to right will have a negative sign.

As the graphic shows, the **rise** is the vertical distance between the two points (the difference in the *y* values) and the **run** is the horizontal distance between them (the difference in the *x* values). To find the slope of any line using the formula, designate one point 1 and the other 2. We've called (2, 8) point 1 and (11, 2) point 2. (It doesn't matter which is which, but once you choose, stick with it.)

Here is the formula for calculating slope:

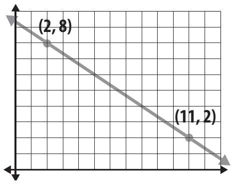
$$\text{slope } (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{or } \frac{\Delta y}{\Delta x}; \text{ or } \frac{\text{rise}}{\text{run}}$$



Example: Find the slope of $\overline{AB}$.

1. Look at the diagram. Determine the coordinates for each point.
2. Substitute the values into the formula.
3. Simplify for slope.



$x_2 = 11$ and $y_2 = 2$; $x_1 = 2$ and $y_1 = 8$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-8}{11-2} = \frac{-6}{9} = -\frac{2}{3}$$

The slope of this line is $-\frac{2}{3}$.

(Note the negative sign. The line is moving downward.)

The slope of a line is always the same, no matter which points are used to find it.

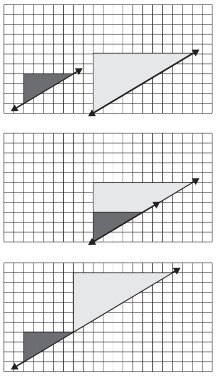
Help Pages

Expressions and Equations 8.EE.1 – 8.EE.8

Graphing — Slope (continued)

Example: We can use what we know about similar triangles to see slope. Compare the slopes of the two triangles.

1. Compare the two triangles. Note that they are similar. (They have the same angles and the side-side ratios are the same.)
2. If we superimpose the two triangles, you can see that the slope of each hypotenuse is the same.
3. Now, we will move the larger triangle until the hypotenuses of both triangles form one line. We know that the slope of each hypotenus is the same. This confirms that the slope of a line at all points is the same.



Using the formula, you can find the slope of a line without the graph; all you need are the coordinates for any two points on the line.

Example: The coordinates (5, 6) and (9, 10) are on the same line. Find the slope.

1. Choose which coordinate will be Point 1 and which will be Point 2.
2. Substitute values into the formula.
3. Solve.

Let (5, 6) be Point 1 and (9, 10) be Point 2.

$$\textit{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

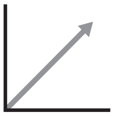
$$= \frac{10 - 6}{9 - 5}$$

$$\textit{slope} = \frac{4}{4} = 1$$

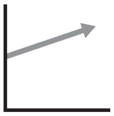
As previously, mentioned, a slope can be negative or positive. A positive slope goes up and to the right. A negative slope goes down and to the right.

Example: Choose the line with the negative slope.

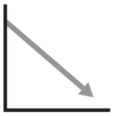
A)



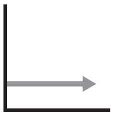
B)



C)



D)



- Graph C shows a negative slope.

A negative slope is calculated just like a positive slope. It's very important that you pay close attention to the signs of the coordinates.

Example: Use the formula to find the slope of the line passing through points (1, 8) and (5, 4).

1. Choose which coordinate will be Point 1 and which will be Point 2.
2. Substitute values into the formula.
3. Solve.

Let (1, 8) be Point 1 and (5, 4) be Point 2.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - 8}{5 - 1}$$

$$\text{slope} = \frac{-4}{4} = -1$$

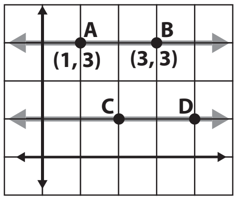
A straight horizontal line has a slope of 0. Look at line $\overline{AB}$ on the the graph. Notice that the *y*-values of both points are the same. This will be true of any points on the same horizontal line. Because of this, $y_2 - y_1$ will always be zero, making the slope equal to zero.

Example: Find the slope of the line passing through the points C and D.

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$= \frac{1 - 1}{4 - 2} = \frac{0}{2}$$

$$\text{slope} = 0$$



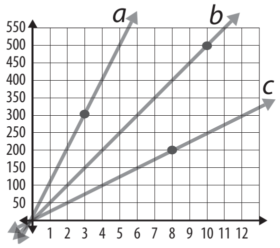
Help Pages

Expressions and Equations 8.EE.1 – 8.EE.8

Graphing — Slope (continued)

Remember, a unit rate is a ratio with a denominator of 1. On a graph, the **unit rate is the same as the slope**. Study the graph.

Example: If a car is traveling at a constant speed of 50 mph, its motion can be represented by the equation $y = 50x$, where 50 is the unit rate. Which line on the graph has a slope of 50?



1. Remember, to determine the unit rate for a line, choose a point on the line. (It is best to choose a point on the grid where x and y meet.)
2. Write the coordinates for this line as a fraction. $\left(\frac{\Delta y}{\Delta x}\right)$
3. Reduce the fraction so that the denominator is one.

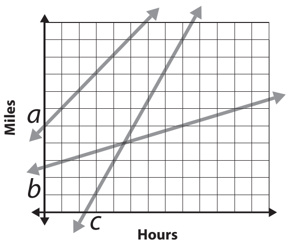
Line $a: \frac{300}{3} = \frac{100}{1}$

Line $b: \frac{500}{10} = \frac{50}{1}$ ✓

Line $c: \frac{200}{8} = \frac{25}{1}$

One way to find out about the unit rate is to study the slope. The line with the steepest slope has the highest unit rate.

Example: Which line has the highest unit rate? If the lines represent 3 cars, which is driving fastest?



Which line has the steepest slope? This will have the highest unit rate and be the fastest car.

- Line c has the steepest slope which means it has the highest unit rate. Therefore, the car represented by Line c is the fastest.

Remember that the same situation, such as a moving car, can be described in a variety of ways: by using a graph, an equation, a data table, or just words. We compared the speeds of a group of cars by studying a graph.

We can make similar comparisons by studying the graph of one car and the equation for another.

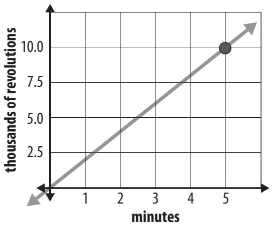
Example: A mechanic compares the engines of two cars. Engine A's motion is described by an equation, and Engine B's motion is described by a graph. Compare the two cars by comparing the graph to the equation. Find the unit rate of both engines in revolutions/minute. Which one has a greater number of RPM?

- The unit rate of Engine A is 2,200 RPM.
- The unit rate for Engine B is $\frac{10,000}{5} = 2,000$ RPM
- Engine A has the greater number of RPM.

Engine A

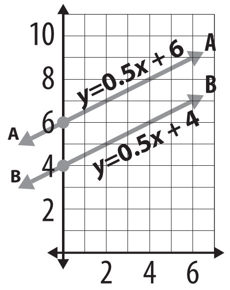
$y = 2,200x$,
where y = revolutions
and x = minutes

Engine B



Some lines on a coordinate plane do not pass through the origin. Study the example. The lines on the graph represent the growth of two plants growing at the same rate. Notice that both lines have the same steepness, or slope. But they differ in another way.

Example: At the beginning, one plant was 4 inches tall and the other was 6 inches tall. Which plant began at 6 inches tall? How do you know?



- Plant A began at 6" tall.
- To represent the beginning of the experiment, it must refer to the point where the line crosses the y-axis.

Help Pages

Expressions and Equations 8.EE.1 – 8.EE.8

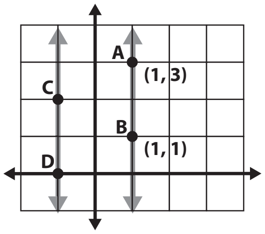
Slope-Intercept Equation of a Line

A vertical line has a slope that is undefined. Look at line $\overline{AB}$ on the graph. Notice that the x-values of both points are the same. This will be true of all points on the same vertical line. Because of this, $x_2 - x_1$ will always be zero, making the denominator of the slope formula equal to zero. Remember, division by zero is undefined.

Example: Find the slope of the line passing through the points C and D.

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{0-2}{-1-(-1)} = \frac{-2}{0}$$

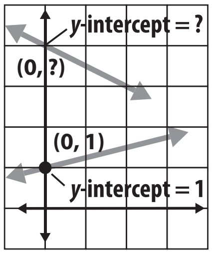
The slope is undefined.



Another characteristic of any line is the ***y*-intercept**. It is the point where a line crosses the y-axis; its coordinates are always (0, y).

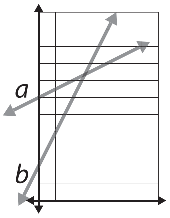
Example: The y-intercept of the bottom line is 1. What is the y-intercept of the top line?

- The y-intercept of the top line is 4.

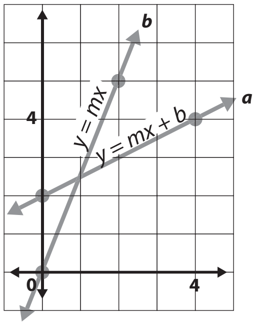


Example: Which line has a higher y-intercept and which has a steeper slope?

- Line a has a higher y-intercept and Line b has a steeper slope.



Remember that a line passing through the origin can be represented by the equation $y = kx$, where k is the unit rate. Notice that for line b in the graph, the y-intercept is zero. **For any line passing through the origin, the *y*-intercept is zero.** Also, we have seen that the unit rate corresponds to the slope (m). If we replace k with m in the equation above we get a new equation. **Any line passing through the origin, has the equation, *y* = *mx*, where *m* represents the slope of the line.** For a line that does not pass through the origin, the y-intercept is not zero. Because of this, the equation must be adjusted. **All lines can be defined by an equation *y* = *mx* + *b*, where *m* is the slope and *b* is the *y*-intercept.** This equation is called the **slope-intercept form**.



Example: Look at the graph. Find the slope and the y-intercept of line b. Write the equation for the line.

$$m = \frac{5-0}{2-0} = \frac{5}{2}; b = 0; \text{ Equation: } y = \frac{5}{2}x + 0 \text{ or } y = \frac{5}{2}x$$

Example: Give the equation for a line with a slope of 5 and a y-intercept of −9.

$y = 5x - 9$

Example: What is the slope and the y-intercept for a line if its equation is $y = 2x + 7$?

slope = 2 and y-intercept = 7

Example: Give the equation for the line with a y-intercept of 4 that passes through points (2, 3) and (4, 2). (Hint: Use the formula to calculate slope.)

$$\text{slope} = \frac{3-2}{2-4} = \frac{1}{-2} = -\frac{1}{2}$$

$$y = -\frac{1}{2}x + 4$$

Help Pages

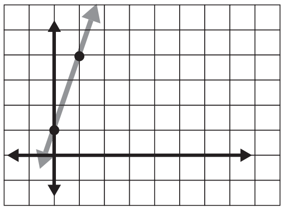
Expressions and Equations 8.EE.1 – 8.EE.8

Graphing a Line

If you know the equation, you can graph the line on a coordinate plane. Consider the equation $y = 3x + 1$.

1. Find the slope and the *y*-intercept.
2. Begin with the *y*-intercept (0, 1) and draw a point.
3. The slope is 3. This tells us that the rise is 3 and the run is 1. From the *y*-intercept, count up 3 and over 1 to the right, and make another point.
4. Draw a line to connect the points.

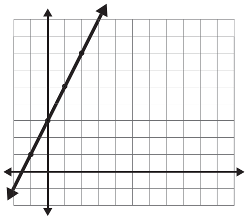
slope = 3 or $\frac{3}{1}$; *y*-intercept = 1

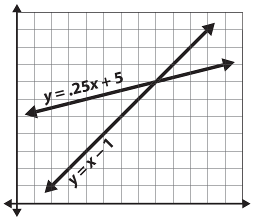


Graphing Solutions for Equations

Most linear equations have many possible solutions. (Although we have seen that some have no solutions at all.) Consider the equation $y = 2x + 3$. You know that there are many pairs of points that will make the equation true. The table represents just a few of the solutions to the equation. **Graphing the equation, however, shows all possible solutions.** Every point on the line is a solution to the equation. That means the line represents the solution set.

$y = 2x + 3$	
x	*y*
-1	1
1	5
0	3
2	7
?	?
?	?



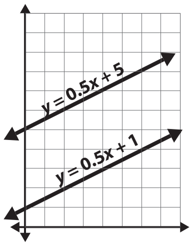


Sometimes we show two (or more) lines on the same grid. Study the graph. Each line represents a different linear equation. The lines intersect at one point. This point (8, 7) is the only solution for both equations — it makes both equations true! Verify this by substituting the *x*-value into each equation and solving for *y*.

$y = 0.25(8) + 5$ $\quad y = 8 - 1$
$y = 2 + 5$ $\quad y = 7$
$y = 7$

When we look for a common solution to two or more equations, it is called **solving a system of equations**. We have just seen that one method for solving a system of equations is to graph them, and see which point they have in common.

Not every system of equations has a common solution. On a graph, these equations would have no shared points. Think about what the graph of this type of system will look like. Remember, lines that do not intersect are called parallel lines.



If two lines are parallel, they have no points in common. So, the system of equations has **no solutions**. Study the two parallel lines. Note that they have the same slope; however, their *y*-intercepts are different. **The graphs of two linear equations will be parallel if they have the same unit rate (same slope) and different *y*-intercept**. Remember, in the equation, the unit rate is the coefficient next to *x* in a linear equation. Which two equations represent parallel lines?

A) $y = 3x + 1$
B) $y = 9x + 1$
C) $y = 3x - 10$
D) $y = -3x + 1$

A and C; both have the same coefficient of 3. This means they each have a slope of 3 and are parallel to each other.

If two linear equations are exactly the same, they will have an infinite number of solutions because any input will have the same output for both equations. Which two equations have an infinite number of solutions?

A) $y = 4x + 1$
B) $y = 4x \times 1$
C) $4x = y + 1$
D) $y - 1 = 4x$

A and D. If you solve D for *y*, you get $y = 4x + 1$. Because the equations are the same, they will have infinitely many solutions.

Systems of equations in which the lines are neither identical nor parallel have exactly one solution. That solution is the point of intersection.

Help Pages

Expressions and Equations　8.EE.1 – 8.EE.8

Solving Systems of Equations: Substitution

Consider the following system.　A) $y - x = 15$　and　B) $y = 2x - 5$

Note that there are two variables in each equation. The **substitution method** involves choosing one of the two equations (you may start with either equation), isolating a variable in that equation, and then substituting that answer into the second equation.

Equation B	Equation A	Equation B
Step ❶ $y = \boxed{2x - 5}$	$\boxed{y} - x = 15$	$y = 2\boxed{x} - 5$
Step ❷	$(2x - 5) - x = 15$	$y = 2(20) - 5$
	$2x - x - 5 = 15$	$y = 40 - 5$
	$x - 5 = 15$	$y = 35$
	Step ❸ $x = \boxed{20}$	Step ❹

1. The first step is to isolate a variable in one of the equations. In Equation B, the y variable is already isolated so we will start with this equation. This equation tells us that $y = 2x - 5$.
2. Because $y = 2x - 5$, we can substitute $2x - 5$ into the second equation (Equation A) in place of y.
3. The second equation (A) now has one variable (in this case x). Solve this equation. We now know the value of x for both equations.
4. Now, put this x-value back into either equation to find the value of y (35).
5. The solution for the system is (20, 35).

Solving Systems of Equations: Elimination

Consider the following system.　A) $2x + 3y = 12$　and　B) $4x - y = 10$

The **elimination method** can be used to solve any system. The goal is to add (or subtract) the equations in order to eliminate one of the variables.

$$\begin{array}{r} 2(2x + 3y = 12) \\ \underline{4x - y = 10} \\ 4x + 6y = 24 \\ \underline{-(4x - y = 10)} \\ 7y = 14 \\ y = 2 \end{array}$$

$$\begin{array}{r} 2x + 3(2) = 12 \\ 2x + 6 = 12 \\ 2x = 6 \\ x = 3 \end{array}$$

1. By multiplying the first equation by 2, and then subtracting, you can see that x will cancel out. We were able to isolate and find the value of y.
2. Now, put this y-value into either equation to find the value of x.
3. We know that $x = 3$ and $y = 2$. The solution for the system is (3, 2).

Functions　8.F.1 – 8.F.5

Identifying Functions

In the equation below, you recognize x as the independent variable and y as the dependent variable.

$$y = x + 1$$

This is because the value of y depends on the value of x. Another way to say this is that ***x* is an input and *y* is an output**. Consider gasoline. You put a certain amount of gasoline into your car (your input) and are then able to drive a certain distance (your output).

Example: Look at the equation. Which letter is the input and which is the output?

$$y = 2x + 7$$

x is the input and y is the output.

A **function is a relationship in which each input value matches up with exactly one output value**. In other words, if you are given an equation such as $y = 2x + 5$, there must only be one possible y value for any x you can put in. If any x value can yield more than one y value, the relationship is not a function. Using this information, choose the set of data that does not represent a function.

A) $y = 2x + 5$

x	y
–1	3
0	5
1	7
2	9

B) $y = x - 7$

x	y
–1	–8
0	–7
1	–6
2	–5

C) $y = -9x$

x	y
–1	9
0	0
1	–9
2	–18

D) $y^2 = x$

x	y
1	–1
1	1
0	0
16	4

The correct answer is D.
The x-values (inputs) each yield two y-values (outputs).

Help Pages

Functions 8.F.1 – 8.F.5

Identifying Functions (continued)

$y^2 = x$
$y^2 = 4$
$y = 2 \text{ or } -2$

An example of an equation that is not a function is $y^2 = x$. As you can see from the example at the right, the output y^2 has both a positive and a negative value. An input of 4 yields two possible outcomes, 2 and −2. **It may be helpful to remember that any equation in which the output is squared (y^2) is not a function**.

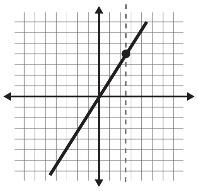
Example: Choose the equation that is not a function.

A) $y = 2x + 1$ B) $y^2 = 3x - 1$ C) $y = x^2 + 5$ D) $y = -x$

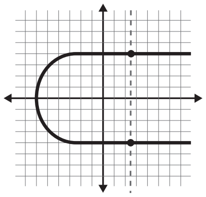
The correct answer is B. Equation B contains y^2, so that any input value is bound to yield 2 outputs.

A graph can be used to determine if an equation is a function. Consider the following graphs. Look at Graph A. Trace any vertical line and you will note that it crosses the line of the equation only one time. For example, you will see that a vertical line at $x = 2$ will cross the line once, at y = 4. This confirms that this equation is a function. On the other hand, look at Graph B. Draw a vertical line through the graph and you are likely to see it cross the line at two points. For example, a path drawn at $x = 2$ crosses the line at $y = 4$ and $y = -4$. This means that the input value of 2 has more than one output value. If a vertical line drawn on the graph crosses the line of the equation twice, then the equation is not a function.

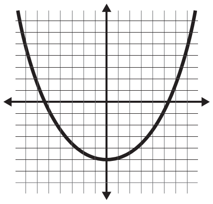
Graph A



Graph B



Example: Determine whether the graph represents a function.



The graph does represent a function. There is no x-value that corresponds to more than one y-value.

Comparing Properties of Functions

Functions can be expressed in many forms. These include graphs, tables, equations, and verbal descriptions. Sometimes it will be necessary to compare the properties of functions such as slope or y-intercept. Instructions for how to calculate these properties can be found in the "Expressions and Equations" portion of the *Help Pages.*

Example: A and B are linear functions. Which function has the greater slope?

Graph A

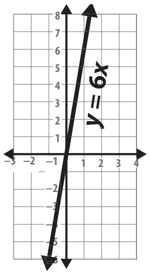


Table B

	x	y	
	-1	-3	
1	0	2	5
1	1	7	5
1	2	12	5

The slope of the graph is 6 (see page 298).
The slope of the line represented by the table is 5 ($\Delta y/\Delta x$).

Graph A has the greater slope.

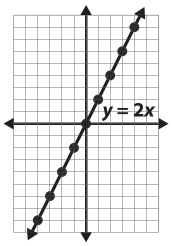
Help Pages

Functions 8.F.1 – 8.F.5

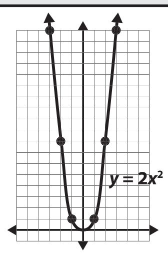
Determining if a Function is Linear

To say that a function is linear is to say that the rate of change for the function is constant.

Using a graph, we can recognize a linear function quite easily. Consider the graphs to the right. As the name implies, the graph of a linear function will result in a straight line. Functions that don't result in a straight line are not linear functions.



linear function



nonlinear function

You can also determine if a function is linear from a table. Remember that linear means the rate of change is constant. If the function is linear, the rate of change for the *x* value and the rate of change for the *y* value must both be constant. If the change is constant, the function is linear. Consider the tables below.

In Table 1, we can see that the change in *x* is constant (all values change by 1), as well as the change in *y* (all values change by 2.)

Table 1 displays a linear function.

In Table 2, the change in *x* is constant. (All values change by 2.) However, the change in *y* is <u>not</u> constant. (The change varies.)

The function in Table 2 is <u>not</u> a linear function.

Table 1

$y = 2x + 7$	
1	9
2	11
3	13
4	15

$2 - 1 = 1$
$3 - 2 = 1$
$4 - 3 = 1$

$11 - 9 = 2$
$13 - 11 = 2$
$15 - 13 = 2$

Table 2

$y = x^2 + 7$	
2	11
4	23
6	43
8	71

$4 - 2 = 2$
$6 - 4 = 2$
$8 - 6 = 2$

$23 - 11 = 12$
$43 - 23 = 20$
$71 - 43 = 28$

Writing Linear Equations

The slope-intercept form of a linear function is $y = mx + b$. To complete this equation, we must know two things:

- *m* — the slope, or rate of change
- *b* — the *y*-intercept, or starting point

Writing linear equations from descriptions

Example: Kaylyn and Jorge start the morning at their lemonade stand with 5 L of lemonade and sell $\frac{1}{2}$ L each hour.

1. To find the slope (m) look for the change being described. From the description, we know that the amount of lemonade goes down by $\frac{1}{2}$ each hour. The slope (*m*) is $-\frac{1}{2}$.
2. The y-intercept is the point at which the line crosses the *y*-axis; in real-life examples, the *y*-intercept can be thought of as the starting point. The description tells us they began the day with 5 L of lemonade. The *y*-intercept is 5.
3. The equation is $y = -\frac{1}{2}x + 5$ or $y = -0.5x + 5$.

Writing linear equations from tables

Example: Write the equation for a line using the information in the table.

x	*y*
−1	−2
0	1
1	4
2	7

1. Use the table to find the slope of the function. Because we know it's a line, any points will work. We will use the points (1, 4) and (2, 7). (The slope of this function is 3.)
2. Find the *y*-intercept. We know that the *y*-intercept is the *y*-value when the line crosses the *y*-axis, or the value of *y* when $x = 0$. Use the table to find the *y*-value when $x = 0$. For this function, *y* is 1 when $x = 0$.
3. The equation is $y = 3x + 1$.
4. To check your work, plug one set of (*x*, *y*) values from the table into the equation.

Step ❶

$$\text{slope} = \frac{\Delta y}{\Delta x}$$

$$\frac{7-4}{2-1} = \frac{3}{1} = 3$$

Step ❷

x	*y*
−1	−2
0	1
1	4
2	7

Step ❹

$$y = 3x + 1$$
$$-2 = 3(-1) + 1$$
$$-2 = -2$$

Our equation is correct!

Help Pages

Functions

8.F.1 – 8.F.5

Writing Linear Equations (continued)

Writing linear equations from two given points

Example: Write the equation for a line if points (2, 5) and (3, 7) are both on the line.

1. Find the slope of the function.
2. Find the y-intercept (b).
   Our equation is now $y = 2x + b$. To solve for b, we will fill in one set of (x, y) values from our given points.
3. The equation is $y = 2x + 1$.
4. To check your work, plug the (x, y) values from the other given point.

Step ❶ $\frac{7-5}{3-2} = \frac{2}{1} = 2$

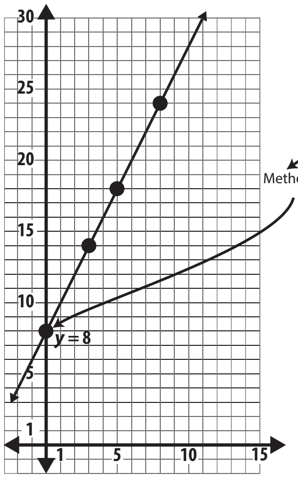
Step ❷
$y = 2x + b$
$5 = 2(2) + b$
$5 = 4 + b$
$1 = b$

Step ❹
$y = mx + b$
$7 = 2(3) + 1$
$7 = 6 + 1$
$7 = 7$

Writing linear equations from graphs

Example: Determine the equation for the line.

1. Choose two sets of points from the graph to determine slope. We will use (5,18) and (3,14).
2. Determine the y-intercept.
   - One way is to read the graph. On this graph, we see that the y-intercept is 8.
   - You can also calculate the y-intercept. This can be helpful when the line does not cross the x-axis at a grid point. Fill in any set of points from the line and solve for b.
3. The equation is $y = 2x + 8$.
4. To check your work, plug in a set of (x, y) values from any point and see if the equation is true.



Step ❶ slope $= \frac{\Delta y}{\Delta x} = \frac{18-14}{5-3} = \frac{4}{2} = 2$

Step ❷

Method 1

Method 2
$y = mx + b$
$18 = 2(5) + b$
$18 = 10 + b$
$8 = b$

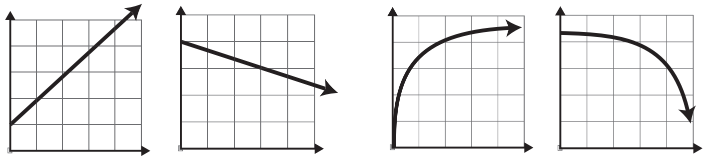
Step ❹
$y = 2x + 8$
$24 = 2(8) + 8$
$24 = 16 + 8$
$24 = 24$
Our equation is correct!

Describing Functions

Graphs and the functions that they illustrate describe a relationship between two quantities. Even without looking at numbers, there is a great deal that we can learn from a graph.

The slope of a graph can be positive or negative. When the slope is positive, the function is increasing. When the slope is negative the function is decreasing. The graphs below illustrate some possibilities.

Linear Functions **Nonlinear Functions**

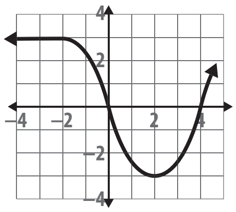


Increasing Decreasing Increasing Decreasing

The slope of a graph can increase <u>and</u> decrease over time. Consider this example.

Example: Describe the graph.

- For x-values to the left of $x = -2$, the function is constant. There is no increase or decrease.
- Between the values of $x = -2$ and $x = 2$, the function is decreasing.
- For values to the right of $x = 2$, the function is increasing.

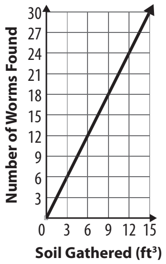


Help Pages

Functions 8.F.1 – 8.F.5

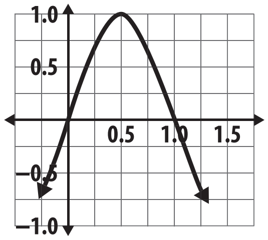
Describing Functions (continued)

Example 1: The graph illustrates the relationship between cubic feet of soil gathered (*x*) and number of worms found (*y*). As the amount of soil increases, does the number of worms increase, decrease, or stay constant? Is this graph linear or nonlinear?



- The slope of the graph is increasing. This tell us that more soil will result in more worms.
- The graph is linear.

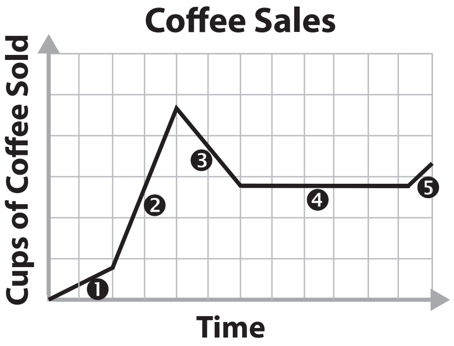
Example 2: Describe the slope of the graph. Is this graph linear or nonlinear?



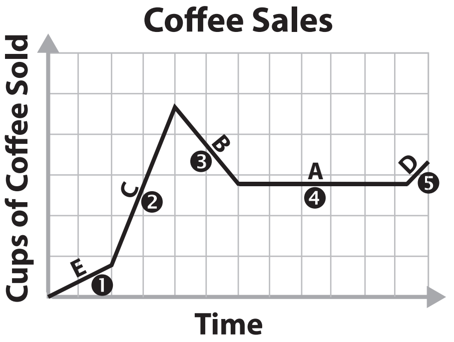
- The slope changes direction at $x = 0.5$.
- For values where $x < 0.5$, the slope is increasing.
- For values where $x > 0.5$, the slope is decreasing.
- The graph is nonlinear.

Example 3: Jennifer sells cups of coffee at a transit station refreshment stand. The graph illustrates the cups of coffee sold over time. Match the descriptions below to their appropriate place on the graph.

A) For a time, the cups of coffee sold remains constant due to a steady flow of sightseers.
B) Sales decrease as most commuters have arrived at work.
C) The greatest number of commuters travel between the hours of 7 and 9 AM . Sales quickly increase during this time.
D) As commuters head back to the station to travel home, coffee sales begin to increases again.
E) For several hours, the sales of coffee increases as commuters travel to work.



Explanation:
A) Section ❹; when the rate is constant, the graph will appear flat.
B) Section ❸; decrease is indicated by a negative or downward slope.
C) Section ❷; Three of the sections of the graph show an increase. This description indicates the quickest increase, so we are looking for the steepest line.
D) and E) Sections ❺ and ❶ respectively. These last two descriptions are both increases. One indicates that it is at the beginning of the day, the other is at the end.



Help Pages

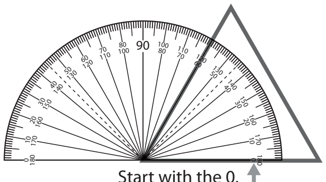
Geometry 8.G.1 – 8.G.9

Using a Protractor

A protractor can be used to measure an angle. The picture at right shows a protractor over a triangle.

Example: Measure the angle of the triangle.

1. The bottom edge of this protractor lines up with that of the triangle. (Protractors may differ.)
2. The center of the edge (where all the lines converge) is placed directly over the triangle's vertex.
3. You can see that the left leg of the triangle lines up exactly with the number 60. This means the measure of the angle is 60°.

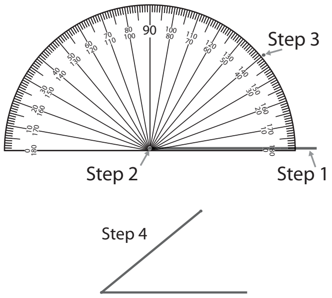


Start with the 0.
Count up and you see the
triangle's side aligns with 60°.

A protractor can also be used to draw an angle.

Example: Draw a 40° angle.

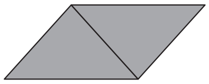
1. First, use the bottom edge of your protractor to draw a straight line. This will form one of the rays of your angle.
2. Your protractor will probably have a hole at the point where all lines intersect. Place this hole over the endpoint of your line and make a dot for your vertex.
3. Now, place a dot at the 40° mark of the protractor. This is the endpoint of your second ray.
4. Finally, remove the protractor and connect the two dots. You now have a forty degree angle. If you need the lines to be a different length when drawing a triangle, you can place a new dot at any point on the line and erase the rest. The angle will remain the same.



Finding the Area of a Triangle

To find the area of a triangle, it is helpful to recognize that any triangle is exactly half of a paralleogram. Because the area of a parallelogram is base × height, the triangle's area is equal to half of the product of the base and the height.

The whole figure is a parallelogram.

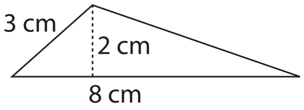


Half of the whole figure is a triangle.

Area of triangle = $\frac{1}{2}$(base × height)

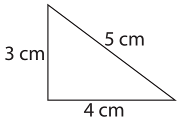
or A = $\frac{1}{2}$bh

Examples: Find the area of the triangles below.



$A = 8\text{ cm} \times 2\text{ cm} \times \frac{1}{2} = 8\text{ cm}^2$

1. Find the length of the base. (8 cm)
2. Find the height. (It is 2 cm. The height is perpendicular to the base.)
3. Multiply them together and divide by 2 to find the area. (8 cm^2)



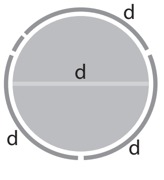
$A = 4\text{ cm} \times 3\text{ cm} \times \frac{1}{2} = 6\text{ cm}^2$

The base of this triangle is 4 cm long.

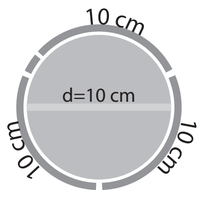
Its height is 3 cm. (Remember, the height is always straight up and down!)

What is pi?

For any circle, π is the ratio of the circumference to the diameter. The value of pi is approximately 3.14 or $\frac{22}{7}$. In other words, the circumference of a circle is equal to a little more than 3 times the diameter. From this we get **the formula for the circumference of a circle: $C = \pi d$ or $C = 2\pi r$.**

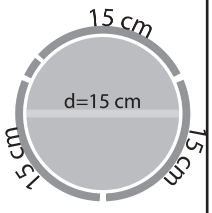


Example 1



$C = \pi d = 31.4$ cm

Example 2



$C = \pi d = 47.1$cm

Help Pages

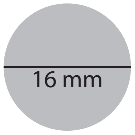
Geometry 8.G.1 – 8.G.9

Circumference of a Circle

The **circumference** of a circle is the distance around the outside of the circle. Before you can find the circumference of a circle you must know either its radius or its diameter. Once you have this information, the circumference can be found by multiplying the diameter by pi (π).

Circumference = C = π × diameter

Example: Use the diameter to find the circumference of the circle below.

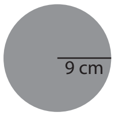


1. Find the length of the diameter. (16 mm)
2. Multiply the diameter by π. (16 mm × 3.14)
3. The product is the circumference.

16 mm × 3.14 = 50.24 mm

Sometimes the radius of a circle is given instead of the diameter. Remember, the radius of any circle is exactly half of the diameter. If a circle has a radius of 3 feet, its diameter is 6 feet.

Example: Use the radius to find the circumference of the circle below.



1. Since the radius is 9 cm, the diameter must be 18 cm.
2. Multiply the diameter by π. (18 cm × 3.14)
3. The product is the circumference.

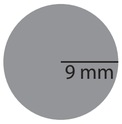
18 cm × 3.14 = 56.52 cm

Finding the Area of a Circle

When finding the **area** of a circle, the length of the radius is squared (multiplied by itself), and then that answer is multiplied by the constant pi (π). $\pi = 3.14$ (rounded to the nearest hundredth) or $\frac{22}{7}$.

Area = π × radius × radius or $A = \pi r^2$

Example: Use the radius to find the area of the circle below.

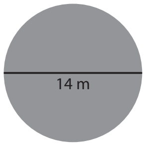


1. Find the length of the radius. (9 mm)
2. Multiply the radius by itself. (9 mm × 9 mm)
3. Multiply the product by pi. ($81\ \text{mm}^2 \times 3.14$)
4. The result is the area. ($254.34\ \text{mm}^2$)

$A = 9\ \text{mm} \times 9\ \text{mm} \times 3.14 = 254.34\ \text{mm}^2$

Sometimes the diameter of a circle is given instead of the radius. Remember, the diameter of any circle is exactly twice the radius. If a circle has a diameter of 6 feet, its radius is 3 feet.

Example: Use the diameter to find the area of the circle below.



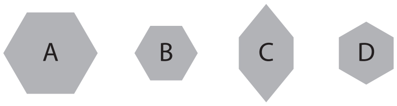
1. Since the diameter is 14 m, the radius must be 7 m.
2. Square the radius. (7 m × 7 m)
3. Multiply that result by π. ($49\ \text{m}^2 \times \frac{22}{7}$)
4. The product is the area. ($154\ \text{m}^2$)

$A = 49\ \text{m}^2 \times \frac{22}{7} = 154\ \text{m}^2$

Congruent Objects — Rotation, Reflection, and Translation

Two shapes are **congruent** if they are exactly the same. This means that their side lengths and angle measures are the same.

Example: Determine which two shapes below are congruent.



1. The sides of shape A are longer that the sides of the other shapes.
2. The angles of shape C are different than the angles of the other shapes.
3. The sides of shapes B and D are the same length <u>and</u> the angles of shapes B and D are the same. Shapes B and D are congruent. We use the **congruent symbol (≅)** to write this B ≅ D.

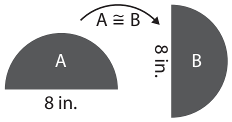
Help Pages

Geometry　　8.G.1 – 8.G.9

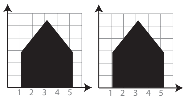
Congruent Objects — Rotation, Reflection, and Translation (continued)

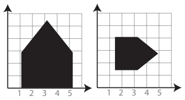
Shapes can be congruent even if they are oriented differently. The orientation of a shape can be changed through **rotation**, **reflection** and **translation**.

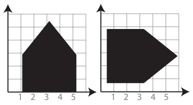
A **rotation** is movement around a point. A and B are congruent, and B has been rotated.

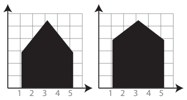


Example: Choose the set that shows congruent shapes where one has been rotated.

A)


B)


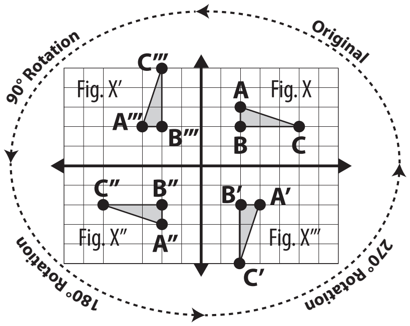
C)


D)


1. Only sets B and C show rotation.
2. In set B, the second object is smaller and, therefore, not congruent.
3. Set C shows two shapes that are congruent and rotated.

If a point of rotation is at the center of a shape, it spins in place (imagine the arrow on a spinner). If the point is outside the shape, it moves in space as it turns (imagine the moon around Earth). We will consider only rotations around the origin.

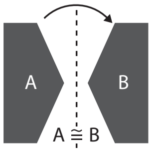
Example: In the figure, the triangle is rotated 90°, 180°, and 270° counter-clockwise around the origin. Examine the table with the coordinates of each point. How do the points in each rotated figure compare to the points in the original triangle?



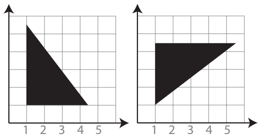
	Original Fig. X	Fig. X′	Fig. X″	Fig. X‴
	0°	90°	180°	270°
A	(2, 3)	(−3, 2)	(−2, −3)	(3, −2)
B	(2, 2)	(−2, 2)	(−2, −2)	(2, −2)
C	(5, 2)	(−2, 5)	(−5, −2)	(2, −5)

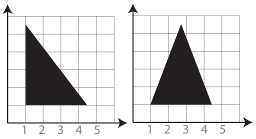
1. 90° rotation: compared to the original, the *x* and *y* coordinates switch places, and the *x* values have the opposite sign.
2. 180° rotation: compared to the original, the *x* and *y* coordinates are the same, except they all have the opposite sign as points A, B, and C.
3. 270° rotation: compared to the original, the *x* and *y* coordinates switch places, and the *y* values have the opposite sign.

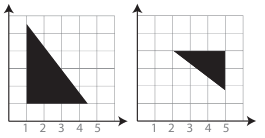
A **reflection** happens when an object is "flipped" over a line of reflection. In the example shown here, A and B are congruent, and A has been reflected (flipped/mirrored), resulting in B.

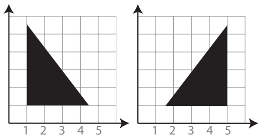


Example: Choose the set that shows congruent shapes where one has been reflected.

A)


B)


C)


D)


1. Set A shows congruent shapes that are rotated.
2. In set B, the shapes are not congruent because the angles are not the same.
3. In set C, the two shapes are not congruent because they are different sizes.
4. In set D, the two shapes are congruent and reflected. This is the correct answer.

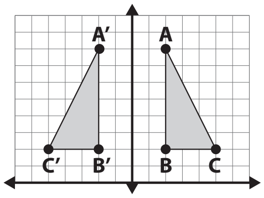
Help Pages

Geometry 8.G.1 – 8.G.9

Congruent Obects — Rotation, Reflection, and Translation (continued)

Here, we'll consider reflections over the *y*-axis as well as reflections over the *x*-axis.

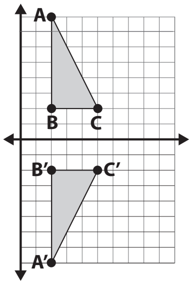
Example: In the figure, the triangle is reflected across the *y*-axis. Examine the coordinates of points A′, B′, and C′. How do they relate to the points in the original triangle?



Reflection *y*-axis	Original Triangle	Reflected Triangle
A	(2, 8)	(−2, 8)
B	(2, 2)	(−2, 2)
C	(5, 2)	(−5, 2)

The points are the same, except the *x* coordinates have the opposite sign as points A, B, and C.

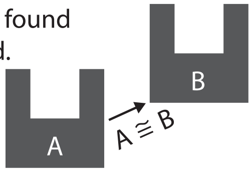
Example: Here, the triangle is reflected across the *x*-axis. Examine the coordinates of points A′, B′, and C′. How do they relate to the points in the original triangle?



Reflection *x*-axis	Original Triangle	Reflected Triangle
A	(2, 8)	(2, −8)
B	(2, 2)	(2, −2)
C	(5, 2)	(5, −2)

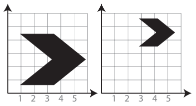
The points are the same, except the *y* coordinates have the opposite sign as points A, B, and C.

A **translation** occurs when a shape is moved. The shape will look exactly the same but it will be found in a new location. In the example shown here, A and B are congruent, and A has been translated.

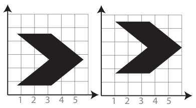


Example: Choose the set that shows congruent shapes where one has been translated.

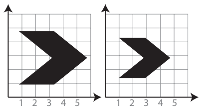
A)



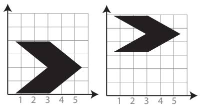
B)



C)



D)

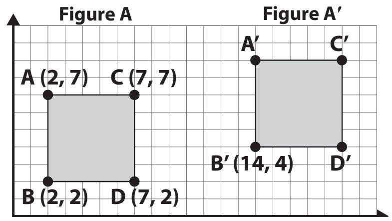


Set B is the only set showing congruent shapes.

A translation (a slide) can be done very precisely by using a coordinate plane. On the graph below, Fig A′ was formed by a translation of Fig. A.

To determine the precise translation, choose corresponding points in each figure. For example, compare points B and B′. How many vertical spaces and horizontal spaces did point B move in the translation? (Remember, on a coordinate plane, a horizontal change represents a change in the *x*-value; a vertical change represents a change in the *y*-value .) Point B moves to the right by 12 spaces and up by 2 spaces. This means the *x*-value changes by +12 and the *y*-value changes by +2. The coordinates for B′ are (2 + 12, 2 + 2) or (14, 4).

Example: Find the coordinates of the other three vertices in Fig. A′.



Vertex A′ is located at (14, 9).
Vertex C′ is located at (19, 9).
Vertex D′ is located at (19, 4).

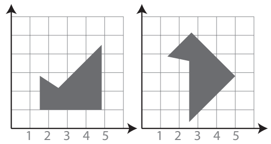
Help Pages

Geometry 8.G.1 – 8.G.9

Congruent Objects — Rotation, Reflection, and Translation (continued)

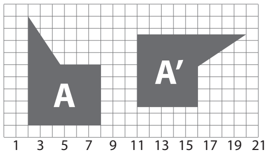
At times, shapes will undergo more than one change. It is possible to have a rotation and a reflection, or a translation and a rotation, or any other combination of these changes.

Example: Study the shapes. The two shapes are congruent. Determine if the two shapes have been rotated, reflected, translated, or some combination of these.



This shape has been reflected and rotated.

Example: Study the shapes. Determine if the shapes are congruent, and if so, what transformations have occurred.



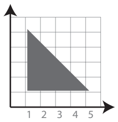
The side lengths and angle measurements of the two shapes are the same, so they are congruent.

The second shape was formed by a rotation (90° clockwise) of the first, and a translation (change in position).

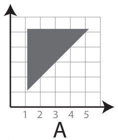
Similar Objects and Dilations

Congruent shapes have the exact same side lengths and angle measures. **Similar shapes have exactly the same angles, but different side lengths.** Two different-sized squares are examples of similar shapes.

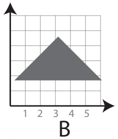
Example: Look at the triangle below. Decide which triangle is similar but <u>not</u> congruent to the first.



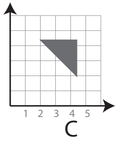
Original



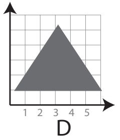
A



B



C

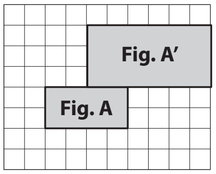


D

1. Figure A is congruent to the original.
2. Figure B is congruent to the original.
3. Figure C has the same angles but different side lengths. Therefore, Figure C is similar.
4. The angles of Figure D differ from the original. It is neither congruent nor similar.

A **dilation is a transformation in which a shape is scaled up or down**. The new shape is not congruent to the original but is <u>similar</u>. All of the side lengths of the tranformed figure will relate to the original figure by the same ratio.

Example: Determine whether the shapes are similar. If not, write *not similar*. If they are, describe the transformations that occurred.

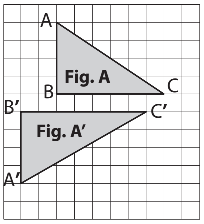


The two shapes <u>are</u> similar.

The corresponding side lengths of both rectangles have a ratio of 2:3.

The image was moved up and to the right.

Example: Determine whether the shapes are similar. If not, write *not similar*. If they are, describe the transformations that occurred.



The two shapes <u>are not</u> similar.

The ratio for $\overline{AB}$ to $\overline{A'B'}$ (1:1) is not the same as the ratio for $\overline{BC}$ to $\overline{B'C'}$ (6:7) .

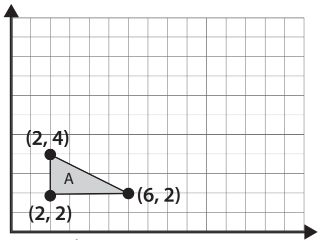
Help Pages

Geometry 8.G.1 – 8.G.9

Similar Objects and Dilations (continued)

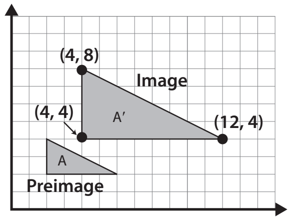
You can draw a dilated figure by using the points on the coordinate plane.

Example: Scale the triangle by a factor of two.



1. Multiply each set of coordinates by the given scale factor.
2. Plot the points for the new coordinates.
3. Connect the points to complete the new image.

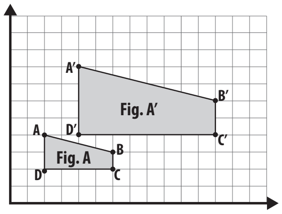
Original Coordinates	Scale Factor	New Coordinates
(2, 2)	2	(4, 4)
(2, 4)		(4, 8)
(6, 2)		(12, 4)



The **scale factor** of a dilation can be determined by comparing two corresponding side lengths (the bases of two similar triangles, for example). Note: scale factor will always be positive.

Example: Fig. A′ is a dilation of Fig. A. Find the scale factor by which Fig. A was dilated.

1. Measure one side length per figure.
2. Divide length of the image (Figure A′), by the length of the preimage (Figure A) to find the scale factor.

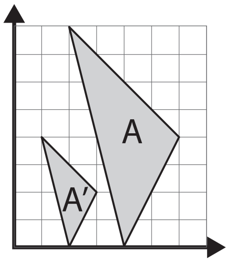


1. We will use the grid to measure lines AD and A′D′. $\overline{AD}$ measures 2 units; $\overline{A'D'}$ measures 4 units.
2. Dividing 4 by 2 is 2. The scale factor is 2.

Coordinates can also be used to determine the scale factor.

Example: A dilation has reduced the size of A and created A′. Determine the scale factor of the dilation by using the coordinates.

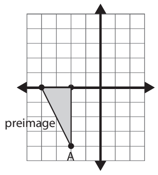
1. Find the coordinates for Figures A and A′.
2. Compare the coordinates for a given point on Figure A with those of the same point on Figure A′.

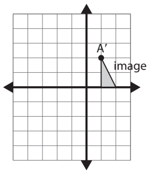


Original Coordinates	New Coordinates
(2, 8)	(1, 4)
(4, 0)	(2, 0)
(6, 4)	(3, 2)

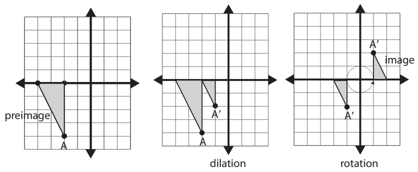
1. The table shows the coordinates.
2. The coordinates for Figure A′ are $\frac{1}{2}$ of the coordinates for Figure A.
3. The scale factor is $\frac{1}{2}$.

Example: Figure A′ is similar to Figure A. Describe one possible sequence of transformations.

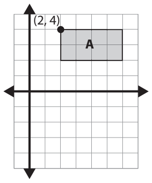


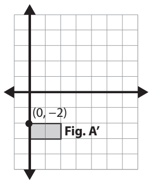


As the graphic to the right shows, one possible sequence may be this:
1. The image has been dilated by a factor of $\frac{1}{2}$.
2. The image was then rotated 180°.

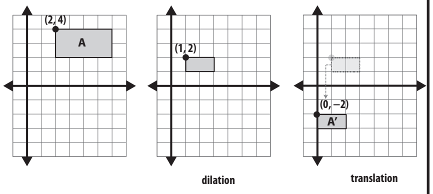


Example: Figure A′ is similar to Figure A. Describe one possible sequence of transformations.





As the graphic to the right shows, one possible sequence may be this:
1. The image has been dilated by a factor of $\frac{1}{2}$.
2. The image was translated (–1, –4), moving one unit to the left and four units downward.



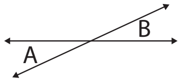
Help Pages

Geometry 8.G.1 – 8.G.9

Types of Angles

Vertical angles are created when two lines intersect. **Vertical angles are always congruent**.

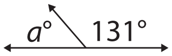
Example: Angles *A* and *B* are vertical angles. If the measure of angle *A* is 25°, what is the measure of angle *B*?



1. Because the angles are vertical, the two angles are congruent.
2. If angle $A = 25°$, then angle $B = 25°$ also.

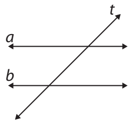
Adjacent angles are created when two lines intersect; they are directly next to one another.

Example: The adjacent angles shown here are supplementary. Give the value of *a*.



1. Because the angles are supplementary, the sum of the two angles is 180°.
2. $180 - 131 = 49$. The measure of angle *a* is 49°.

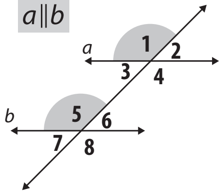
A **transversal** is any line that passes through two lines. Identify the transversal in the graphic below.



Line *t* is the transversal line.

When a transversal passes through two lines, it creates **corresponding angles**. **If the lines are parallel, the corresponding angles are congruent angles** situated the same way relative to the transversal and to their respective parallel lines.

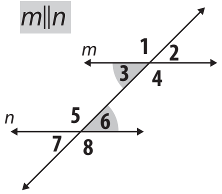
Example: $\angle 1$ and $\angle 5$ are corresponding angles because they are both to the left of the transversal and above their respective lines. Find another pair of corresponding angles.



$\angle 2$ and $\angle 6$ are corresponding angles, $\angle 3$ and $\angle 7$ are corresponding angles, and $\angle 4$ and $\angle 8$ are corresponding angles.

When a transversal passes through two lines, it always also creates four interior angles. **If the lines are parallel, alternate interior angles, located on the opposite sides of the transversal between the two lines, are always congruent.**

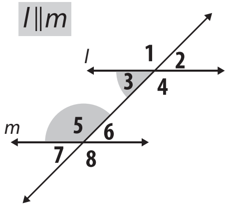
Example: $\angle 3$ and $\angle 6$ are one example of alternate interior angles. Name the other pair.



$\angle 4$ and $\angle 5$ are the other pair of alternate interior angles.

Same side interior angles are located on the same side of the transversal between two lines. **If the lines are parallel, they are always supplementary.**

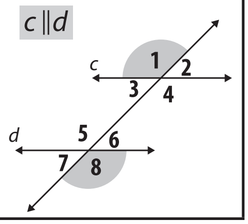
Example: $\angle 3$ and $\angle 5$ are one example of same side interior angles. Name the other pair.



$\angle 4$ and $\angle 6$ are the other pair of same side interior angles.

Alternate exterior angles appear above and below the two lines on the opposite sides of the transversal. **If the lines are parallel, they are always congruent.**

Example: $\angle 1$ and $\angle 8$ are alternate exterior angles. Name the other pair.



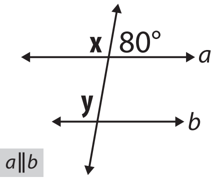
$\angle 2$ and $\angle 7$ are the other pair of alternate exterior angles.

Help Pages

Geometry 8.G.1 – 8.G.9

Types of Angles (continued)

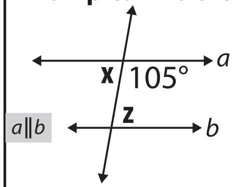
Example: Two parallel lines are intersected by another line. Find the measures of *x* and *y*.



1. Together *x* and the 80° angle form a straight line. We can find angle *x* by subtracting 180 – 80.
2. Angles *x* and *y* are corresponding angles. Therefore, they are congruent.

1. 180 – 80 = 100°, $x = 100°$
2. $x \cong y$; $y = 100°$

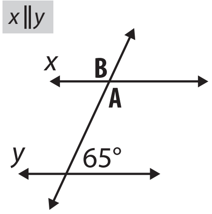
Example: Find the measures of *x* and z.



1. Together *x* and the 105° angle form a straight line. We can find angle *x* by subtracting 180 – 105.
2. Angles *x* and *y* are alternate interior angles. Therefore, they are congruent.

1. 180 – 105 = 75°, $x = 75°$
2. $x \cong z$; $z = 75°$

Example: Find the measures of *A* and *B*.



1. Angles A and the 65° angle are same side interior angles, therefore they are supplementary. The sum of the two angles is 180°.
2. Angles *A* and *B* are vertical angles. Therefore, they are congruent.

1. 180 – 65 = 115°, $A = 115°$
2. $A \cong B$; $B = 115°$

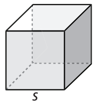
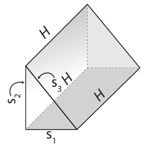
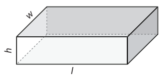
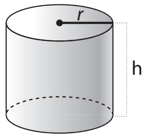
Finding the Area of a Regular Polygon

circle	$A = \pi r^2$	square	$A = s^2$
parallelogram	$A = b \times h$	trapezoid	$A = \frac{(b_1 + b_2) \times h}{2}$ or $\frac{1}{2}(b_1 + b_2) \times h$
rectangle	$A = l \times w$	triangle	$A = \frac{1}{2}(b \times h)$

Solid Figures

Prism — a three-dimensional figure that has two identical, parallel bases and three or more rectangular faces. (There are as many faces as there are sides on the bases.) A prism gets its name from the shape of its base.

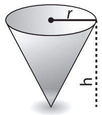
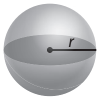
Types of Solids

Figure	Description	Example	Volume	Surface Area
cube	square bases and faces; all edges, faces and vertices are congruent (blocks, number cubes, etc.)		s^3 or $l \times w \times h$	$6s^2$ where s is the side length
triangular prism	a prism with two triangular bases and three rectangular faces (type of prism that bends light and creates a rainbow)		*Bh* area of triangular base × height (*H*) of prism	(areas of two bases) + (areas of 3 faces)
right rectangular prism	a prism with rectangular bases and four rectangular faces (shoe box, refrigerator, pizza box)		$l \times w \times h$	$2(lw + lh + hw)$
cylinder	a smooth shape with a circular base on each end, connected by a curved surface		$\pi r^2 h$ area of circular base × height (*h*) of cylinder	

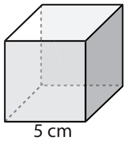
Help Pages

Geometry 8.G.1 – 8.G.9

Solid Figures (continued)

Figure	Description	Example	Volume	Surface Area
cone	a shape with a circular base and one vertex		$\frac{1}{3}\pi r^2 h$	
sphere	all points on the surface are the same distance from the center of the figure		$\frac{4}{3}\pi r^3$	

Example: Calculate the volume and surface area of this cube.

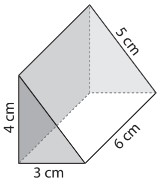


To calculate volume:
1. Because this is a cube, all sides have the same length. In this cube, the side length is 5 cm.
2. For a cube volume (V) = s^3, so V = $5^3 = 125\text{ cm}^3$

To calculate surface area:
1. For a cube, surface area = $6s^2$, so SA = $6(5^2) = 150\text{ cm}^2$

Volume:
$V = 5^3 = 125\text{ cm}^3$
Surface Area:
$SA = 6s^{2\,=}6(5^2) = 150\text{ cm}^2$

Example: Calculate the volume and surface area of this triangular prism.

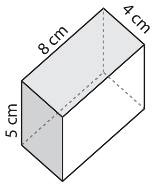


To calculate volume:
1. First, find the area of the triangular base (A = $\frac{1}{2}$bh).
2. Multiply the area by the height of the prism.

To calculate surface area:
1. Find the area for the two triangular bases.
2. Because there are two bases, double this number.
3. Find the area for each rectangular side. All three rectangles have a height of 6 cm. Multiply 6 times each side length.
4. Add the area of the two bases to the area of each rectangle.

Volume:
$A = \frac{1}{2}(3 \times 4) = 6\text{ cm}^2$
$V = 6 \times 6 = 36\text{ cm}^3$
Surface Area:
❶ $A = \frac{1}{2}bh = \frac{1}{2}(3 \times 4) = 6\text{ cm}^2$
❷ $6 \times 2 = 12\text{ cm}^2$
❸ $\begin{cases} 6 \times 3 = 18\text{ cm}^2 \\ 6 \times 5 = 30\text{ cm}^2 \\ 6 \times 4 = 24\text{ cm}^2 \end{cases}$
❹ $12 + 18 + 30 + 24 = 84\text{ cm}^2$

Example: Calculate the volume and surface area of this right rectangular prism.

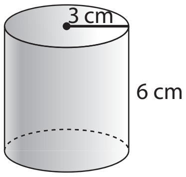


To calculate volume:
The formula for volume of a rectangular prism is $l \times w \times h$.

To calculate surface area:
1. Find the area of three different sides.
2. Add the areas.
3. Because there are two of each, double the areas.

Volume:
$5 \times 8 \times 4 = 160\text{ cm}^3$
Surface Area:
❶ $4 \times 5 = 20\text{ cm}^2$
$8 \times 4 = 32\text{ cm}^2$
$8 \times 5 = 40\text{ cm}^2$
❷ $20 + 32 + 40 = 92$
❸ $92 \times 2 = 184\text{ cm}^2$

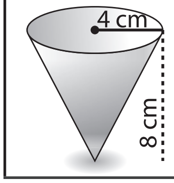
Example: Calculate the volume of this cylinder. Round to the nearest hundredth.



To calculate volume:
The formula for the volume of a cylinder is $\pi r^2 h$.

Volume:
$V = 3.14 \times 3^2 \times 6 = 169.56\text{ cm}^3$

Example: Calculate the volume of this cone. Round to the nearest hundredth.



To calculate volume:
The formula for the volume of a cone is $\frac{1}{3}\pi r^2 h$.

Volume:
$V = \frac{1}{3} \times 3.14 \times 4^2 \times 8 = 133.97\text{ cm}^3$

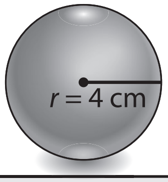
Help Pages

Geometry

8.G.1 – 8.G.9

Solid Figures (continued)

Example: Calculate the volume of this sphere. Round to the nearest hundredth.



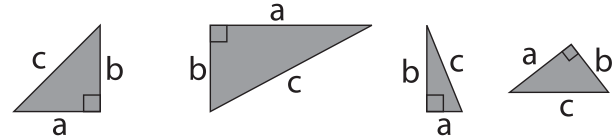
To calculate volume:
The formula for the volume of a sphere is $\frac{4}{3}\pi r^3$.

Volume:
$V = \frac{4}{3} \times 3.14 \times 4^3 = 267.95 \text{ cm}^3$

The Pythagorean Theorem

A **right triangle** is composed of two legs and a hypotenuse. The **legs** are two sides which form a right angle. The **hypotenuse** is always the longest of the three sides and it is always opposite the right angle.

Example: Which letter represents the hypotenuse of each right triangle shown?



Answer: C

The **Pythagorean Theorem** defines the relationship between the lengths of the two legs and the length of the hypotenuse in a right triangle. The equation is $a^2 + b^2 = c^2$, **where *a* and *b* are the lengths of the legs and *c* is the length of the hypotenuse**.

Example: A right triangle has side lengths 12, 16, and 20 cm. Which of the side lengths represents the hypotenuse? Verify that the Pythagorean Theorem is true for this triangle.

1. Determine the hypotenuse.
2. Calculate the value of the hypotenuse squared.
3. Plug the values of the two legs into the equation $a^2 + b^2 = c^2$.
4. Solve the equation for c^2.
5. Compare the value for c^2 to your answer in #2.

1. The hypotenuse is the longest side. In this triangle, the hypotenuse is 20 cm.
2. $20^2 = 400$ cm
3. The two legs of the triangle are 12 cm and 16 cm.
4. $144 + 256 = 400$
5. Our answer from #4 equals our answer from #2. This proves that the Pythagorean Theorem is true for this triangle.

The Pythagorean Theorem can also be used to determine whether or not a triangle is a right triangle. If it is a right triangle, the three side lengths will fit the equation $a^2 + b^2 = c^2$. After substituting the lengths into the Pythagorean Theorem, if the equation is not true, then the triangle is not a right triangle.

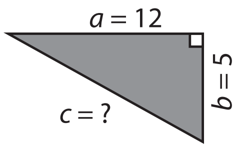
Example: Determine if a triangle with side lengths 7, 8, and 10 is a right triangle.

1. Determine the hypotenuse.
2. Calculate the value of the hypotenuse squared.
3. Plug the values of the two legs into the equation $a^2 + b^2 = c^2$.
4. Solve the equation for c^2.
5. Compare the value for c^2 to your answer in #2.

1. The hypotenuse is the longest side. In this triangle, the hypotenuse is 10 cm.
2. $10^2 = 100$ cm
3. The two legs of the triangle are 7 cm and 8 cm.
4. $49 + 64 = 113$
5. $100 \neq 113$. This is not a right triangle.

If you know that a triangle is a right triangle, the Pythagorean Theorem can be used to find the length of any side.

Example: Find the length of the missing side.



1. Plug the values for a and b into the equation: $a^2 + b^2 = c^2$
2. Solve for c^2.
3. Take the square root of your answer to find *c* (the hypotenuse).

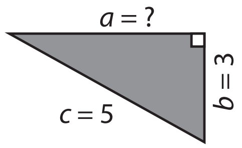
1. $12^2 + 5^2 = c^2$
2. $144 + 25 = 169$
3. $\sqrt{169} = c = 13$

Help Pages

Geometry　8.G.1 – 8.G.9

The Pythagorean Theorem (continued)

Example: Find the length of the missing side.

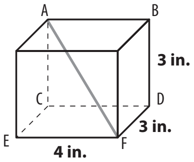


1. Plug the values for b and c into the equation $a^2 + b^2 = c^2$.
2. Solve for a^2.
3. Take the square root of your answer to find a.

1. $a^2 + 3^2 = 5^2$
2. $a^2 + 9 = 25$
   $25 - 9 = a^2$;
   $25 - 9 = 16$
3. $\sqrt{16} = a = 4$

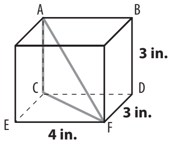
The Pythagorean Theorem can also be used when working with 3-dimensional figures. The trick is finding the right triangles within the figure.

Example: Study the prism. What is the length of the diagonal $\overline{AF}$?



1. Notice that diagonal $\overline{AF}$ is also the hypotenuse of right triangle ACF.
2. We can use the Pythagorean Theorem to solve for $\overline{AF}$. Let $\overline{AC}$ be a and $\overline{CF}$ be b.
3. Find the value of $\overline{AC}$:
   - From the diagram, we see that $\overline{AC}$ is 3 inches (the height of the prism). Plug this value into the equation.
4. Find the value of $\overline{CF}$:
   - To find $\overline{CF}$ we must look for a different triangle. We can see that $\overline{CF}$ is the hypotenuse of ΔCDF.
   - From the diagram, we can see that the legs of ΔCDF ($\overline{CD}$ and $\overline{FD}$) are 4 inches and 3 inches, respectively.
5. Plug these values into the equation to solve for $\overline{CF}$.
6. Go back to the equation from step 2 and plug in the value for $\overline{CF}$. Solve for $\overline{AF}$.

Step ❶



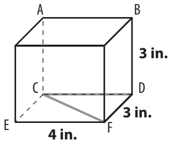
Step ❷

$$c^2 = a^2 + b^2$$
$$\left(\overline{AF}\right)^2 = \left(\overline{AC}\right)^2 + \left(\overline{CF}\right)^2$$

Step ❸

$$\left(\overline{AF}\right)^2 = \left(\overline{AC}\right)^2 + \left(\overline{CF}\right)^2$$
$$\left(\overline{AF}\right)^2 = 3^2 + \left(\overline{CF}\right)^2$$

Step ❹



Step ❺

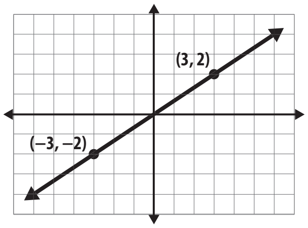
$$\left(\overline{CF}\right)^2 = \left(\overline{CD}\right)^2 + \left(\overline{FD}\right)^2$$
$$\left(\overline{CF}\right)^2 = 4^2 + 3^2$$
$$\left(\overline{CF}\right)^2 = 25$$
$$\sqrt{\left(\overline{CF}\right)^2} = \sqrt{25}$$
$$\overline{CF} = 5 \text{ in.}$$

Step ❻

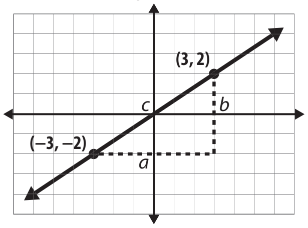
$$\left(\overline{AF}\right)^2 = \left(\overline{AC}\right)^2 + \left(\overline{CF}\right)^2$$
$$\left(\overline{AF}\right)^2 = 3^2 + \left(\overline{CF}\right)^2$$
$$\left(\overline{AF}\right)^2 = 3^2 + 5^2$$
$$\left(\overline{AF}\right)^2 = 34$$
$$\sqrt{\left(\overline{AF}\right)^2} = \sqrt{34}$$
$$\overline{AF} \approx 5.8 \text{ in.}$$

The Pythagorean Theorem can be used to find the distance between two points on a coordinate plane.

Example: Find the distance between (–3, –2) and (3, 2).

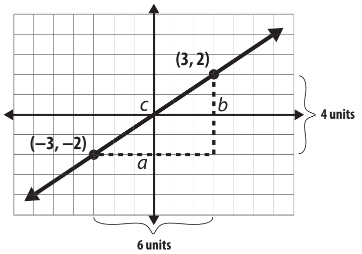


Step ❶



1. Using the grid, draw a horizontal line from point (–3, –2) and a vertical line from point (3, 2) until they meet. This forms a triangle. The hypotenuse of this triangle is the distance we are trying to determine. We can use the equation $a^2 + b^2 = c^2$.
2. Count the units on the grid to find the length of a and b.
3. Plug these values into the equation. Solve for c. Round to the nearest tenth.

Step ❷



Step ❸

$$c^2 = a^2 + b^2$$
$$c^2 = 6^2 + 4^2$$
$$c^2 = 52$$
$$\sqrt{c^2} = \sqrt{52}$$
$$c \approx 7.2 \text{ units long}$$

Help Pages

Geometry 8.G.1 – 8.G.9

The Pythagorean Theorem (continued)

The Pythagorean Theorem can be used to find the distance between two coordinates.

Example: Find the distance between (−2, −2) and (3, 4).

1. First, find the length of the horizontal leg by subtracting the x values.
2. Then, find the length of the vertical leg by subtracting the y values.
3. Plug the values for the leg lengths into the equation $c^2 = a^2 + b^2$ to find the length of the hypotenuse. Round to the nearest tenth.

$$3-(-2)=5 \text{ (} x \text{ values)}$$
$$4-(-2)=6 \text{ (} y \text{ values)}$$

$$\begin{aligned} c^2 &= 5^2+6^2 \\ c^2 &= 25+36 \\ c^2 &= 61 \\ \sqrt{c^2} &= \sqrt{61} \\ c &\approx 7.8 \end{aligned}$$

The Number System 8.NS.1 – 8.NS.2

Multiplication and Division

Operation	Factor (Dividend)	Factor (Divisor)	Product (Quotient)
Multiplication (or Division)	+	+	+
	−	−	+
	+	−	−
	−	+	−

Fractions – Undefined

In a fraction, the denominator cannot be zero. Because of the inverse relationship, you know that $\frac{0}{4}$ or $0 \div 4 = 0$. This does not work with $\frac{4}{0}$ or $4 \div 0$ because $0 \times 0 \neq 4$. **If zero is in the denominator of a fraction, the answer is undefined.**

Rational and Irrational Numbers

A **rational number** is any number that can be expressed as a fraction $\frac{a}{b}$ where a and b are both integers and b is not zero. This includes all fractions and mixed numbers. The following are examples of rational numbers:

- $\frac{56}{7}$ because it is already written in the form of $\frac{a}{b}$
- 48 because it can be written as the fraction $\frac{a}{b}$; $\frac{48}{1}$
- $-6\frac{1}{2}$ because both mixed numbers and negative numbers can be written as $\frac{a}{b}$, $-\frac{13}{2}$

Rational numbers also include all terminating decimals, which are simply decimal numbers that stop at some point such as 0.45823, **and repeating decimals** in which the same number or group of numbers repeat infinitely such as $0.\overline{47}$. The use of bar notation symbolizes a digit or group of digits that repeat indefinitely, in this case, 0.4747474747 to infinity. Both terminating and repeating decimals can be written in the form of $\frac{a}{b}$ where both a and b are integers and b is not zero. The following are more examples of rational numbers:

- 0.57 because it can be written in the form of $\frac{a}{b}$, $\frac{57}{100}$
- $0.\overline{3}$ because it is can be written in the form of $\frac{a}{b}$, $\frac{1}{3}$
- –0.75 because negative decimal numbers can be written in the form of $\frac{a}{b}$, $-\frac{3}{4}$

Example: Circle the rational numbers.

[✓ $\frac{90}{360}$] [✓ 0] $\frac{7}{12}\pi$ [✓ −4,009] [✓ $0.\overline{56}$] 0.5672589… [✓ $-33\frac{1}{3}$]

Help Pages

The Number System 8.NS.1 – 8.NS.2

Rational and Irrational Numbers (continued)

An **irrational number** is a decimal that does not terminate nor does it have a repeating digit or pattern of repeating digits. Because of the continuing nature of irrational numbers, symbols are used to indicate them.

- The symbol **…** (read as "dot, dot, dot") after a series of numbers, indicates that the digits continue indefinitely, but not in a fixed, repeating pattern.
- Pi, π, is irrational; it represents a non-terminating, non-repeating decimal number.
- Often, $\sqrt{\ }$ indicates an irrational number. Some square roots, such as $\sqrt{9}$, which is 3, and $\sqrt{6.25}$, which is 2.5, are rational. But other square roots, like $\sqrt{5}$, which $\approx 2.2360679774\ldots$, are irrational because they neither terminate nor repeat.

The following are examples of irrational numbers: 5.6730692758.... $\sqrt{7}$ $\frac{1}{\pi}$

Example: Circle the irrational numbers.

$0.\overline{87}$ [✓ 2π] $-6\frac{4}{5}$ [✓ 0.849237655…] [✓ $\sqrt{17}$] 5^2

To write a repeating decimal number as a fraction, place the repeat portion as the numerator of a fraction with an equal number of 9s as its denominator. Divide.

Example: Write $0.\overline{7}$ as a fraction.

There is one digit in the numerator, therefore, there will be one 9 in the denominator. $\frac{7}{9}$

Example: Show that $0.\overline{39}$ is rational.

A **rational number** is any number that can be expressed as a fraction $\frac{a}{b}$, where a and b are both integers and b is not zero. $0.\overline{39}$ can be written as $\frac{39}{99}$, therefore it is a rational number.

Example: Show that $0.\overline{125}$ is rational.

$0.\overline{125}$ can be written in the form $\frac{a}{b}$ as $\frac{125}{999}$.

Perfect Squares and Irrational Numbers

The first 5 perfect square numbers are 1, 4, 9, 16, and 25. The square root of a perfect square (e.g. $\sqrt{4}$) is a rational number because the square root of any perfect square is an integer (4 is a perfect square, so $\sqrt{4} = 2$). On the other hand, the square root of a number that is not a perfect square (ie. $\sqrt{5}$), is not so easy to answer. 5 is not a perfect square; $\sqrt{5}$ is an irrational number.

The easiest way to approximate $\sqrt{5}$ or any square root that isn't a perfect square is by using a calculator. If a calculator is unavailable, or if yours doesn't have a square root function, the value can be approximated using a series of repetitious steps: guess, divide, average, divide, average, etc. Continue the process to one place value beyond the designated place value. Use that information for rounding purposes. Square your final answer as a way of checking the square root of the irrational number.

Help Pages

The Number System 8.NS.1 – 8.NS.2

Perfect Squares and Irrational Numbers (continued)

Example: Find the approximate value of $\sqrt{5}$.

The result, after each average, is an approximation of the value of $\sqrt{5}$. Each time you go through the process, you get a better and better approximation. Work until you have one additional place value beyond the one designated. In this case, work to the thousandths place in order to round to the nearest hundredth.

- 5 is between which two perfect squares? (4 and 9) Which of them is it closer to? (4)
- $\sqrt{5}$ is between $\sqrt{4}$ and $\sqrt{9}$. So, $\sqrt{5}$ is between 2 and 3, and is closer to 2. Our guess is 2.
- Divide 5 by 2 to get 2.5.
- Average 2 and 2.5 to get 2.25.
- Divide 5 by 2.25 to get 2.22.
- Average 2.22 and 2.25 to get 2.235.
- Round 2.235 to hundredths place (2.24)
- Check the approximated value by squaring it. ($2.24^2 = 5.0176$)
- $5.0176 \approx 5$

Ratios & Proportional Relationships 7.RP.1 – 7.RP.3

Percent – Simple Interest

Interest is an *amount that is paid* when money is borrowed or an *amount that is earned* when money is lent. The amount that is borrowed or lent is called the **principal**. The principal plus the interest is paid by the borrower to the lender after an agreed upon time.

Simple interest is a percentage of the principal, and it adds up over the time of the loan. Simple interest is calculated using this formula:

principal × rate × time = interest or $prt = i$

For example, if you borrow $500 from a bank at 4% annual interest, $500 is the **principal**, 4% is the **rate**, and one year is the **time**. After one year, you would pay $520. Here is why:

$$\underbrace{(500 \times 0.04) \times 1}_{\text{simple interest}} = 500 + 20 = \$520$$

Example:

Juanita deposits $5,500 in a high-interest CD. The account earns 9% annual interest. If she doesn't add or subtract any money from the account for 3 years, how much will be in the account at the end of that time?

1. First identify the principal, the rate, and the time.
2. Next, find the interest using the formula $prt = i$.
3. Finally, add the interest to the original investment.

Principal = $5,500
Rate = 9%
Time = 3 years

Interest $5{,}500 \times 0.09 \times 3 = \$1{,}485$
Total Value $\$5{,}500 + 1{,}485 = \$6{,}985$

Percent Error

Percent error is a measure of how inaccurate a measured value is by comparing your result to the accepted value. To calculate percent error, use this formula:

$$\left|\frac{\text{accepted value} - \text{your result}}{\text{accepted value}}\right| \times 100 = \%\ \text{error}$$

Example: Sean measured the amount of liquid in a graduated cylinder. He recorded a value of 33.4 ml. The accepted volume was 32.5 ml. Calculate the percent error in Sean's measurement. (Note the absolute value sign.)

$$\left|\frac{32.5 - 33.4}{32.5}\right| \times 100 = 2.76\%$$

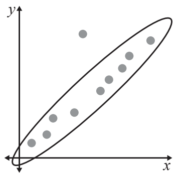
Help Pages

Statistics and Probability 8.SP.1 – 8.SP.4

Scatter Plots

A **scatter plot** is a graph of plotted points that show the relationship between two sets of data graphed as ordered pairs on a coordinate plane.

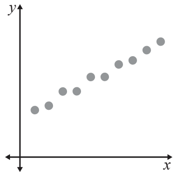
If there is a relationship betwen the two sets of data, the scatter plot will show a cluster. A **cluster** is a collection of points that are close together on a scatter plot.



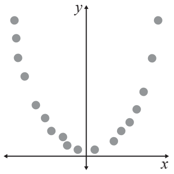
cluster

Scatter plots can be described in several ways:

- They can have a linear association. If a scatter plot is linear, the data points will lie close to a line.
- If a scatter plot is nonlinear, the data points will not lie close to a line; they may be more in the shape of a curve.



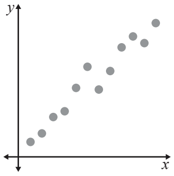
linear association



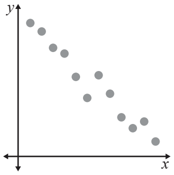
nonlinear association

They can have variable association.

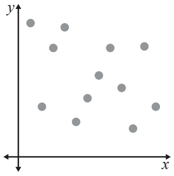
- If a scatter plot has a **positive association**, data points go upward. Or as the ***x*** increases, the ***y*** increases as well.
- If a scatter plot has a **negative association**, data points will go downward. Or as the ***x*** increases, the ***y*** decreases.
- If a scatter plot has **no association**, data points will not form an obvious pattern.



positive association



negative association

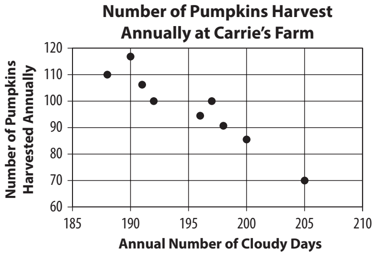


no association

Example: Study the graph. There is a negative association between the annual number of cloudy days and the number of pumpkins Carrie harvests. What does this mean?

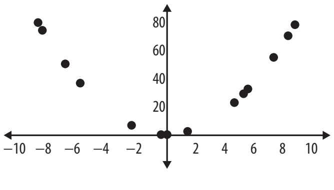
A) One year, Carrie harvested a negative number of pumpkins.
B) As the number of cloudy days increases, the number of pumpkins harvested decreases.
C) As the number of cloudy days increases, the number of pumpkins harvested increases.

The answer is B. The graph has a negative association, so when *x* increases, *y* decreases. In this example, the *x*-axis describes the number of cloudy days, which increases, and the *y*-axis describes the number of pumpkins harvested, which decreases.



Example: The scatter plot shows the relationship between *x* and *y*. Until *x* = 0, the graph decreases. From *x* = 0 on, the graph increases. What kind of association does this depict? Write "linear," "nonlinear," or "no association."

This scatter plot shows a nonlinear association. The data points do not lie in a straight line. It is more in a shape of a curve, so it is nonlinear.



Does the data have any outliers?

A) Yes. Because the graph is not a straight line, there are many outliers.
B) No. Even though the graph is not a straight line, there is still a cluster, and all the data is included in that cluster.
C) Not enough information is given.

The answer is B. There are no outliers in this scatter plot. All of the data points are clustered together.

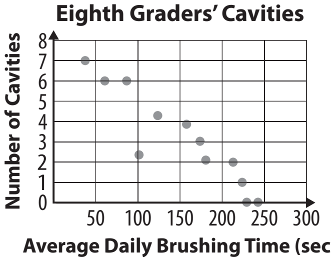
Statistics and Probability

8.SP.1 – 8.SP.4

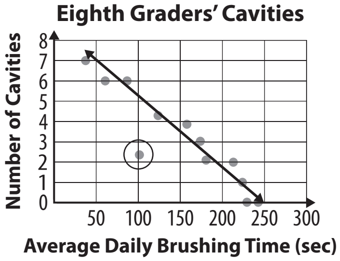
Scatter Plots (continued)

In addition to displaying data that has been gathered, scatter plots can also be used to predict data that has not been gathered. With data that has a linear shape, this is done by **creating a line of best fit**, or **linear model**. A line of best fit is a line that is very close to most of the data points. Points on the graph that lie far outside the line of best fit are called **outliers**.

Example: Study the scatter plot and draw the line of best fit. Circle any outliers.



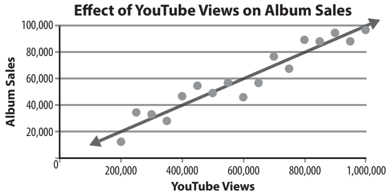
The scatter plot on the right shows a line of best fit drawn in. To draw a line of best fit, place the line so that about one half of the data points are above the line and one half of the points are below the line.



Using the slope of the line of best fit, will help to predict unobserved results.

Example: Study the scatter plot. The equation of the line of best fit is $y = \frac{1}{10}x$. The slope is $\frac{1}{10}$. What does that mean in terms of the data?

A. For every YouTube view, artists sell approximately 10 albums.
B. Artists sell approximately one album for every 10 YouTube views.
C. Each artist sells $\frac{1}{10}$ of an album per year.



Answer: Artists sell approximately one album for every 10 YouTube views. Using the line of best fit, you can see there are 10 YouTube views for every one album sale on the scatter plot.

Example: How many album sales would an artist expect if she received 350,000 YouTube views? Use the equation of the line of best fit to find your answer.

Answer: The equation of the line of best fit is $y = \frac{1}{10}x$. YouTube views are on the *x*-axis, and album sales are on the *y*-axis. To find the number of albums an artist can expect to sell, plug in the given *x* value, (350,000) into the equation. $y = \frac{1}{10}(350{,}000)$; $y = 35{,}000$. An artist can expect to sell 35,000 albums.

Two-Way Tables

Another way to study the relationship between two variables is with a **two-way table**.

Example: There are 292 eighth graders at Davey Middle School. How many are in a sport and in band? How many are in neither?

	in band	not in band	total
in a sport	31	121	31 + 121 = 152
not in a sport	110	30	110 + 30 = 140
total	31 + 110 = 141	121 + 30 = 151	292

Answer: Using the two-way table, you can see there are 31 students who are in a sport and in band. There are 30 students who are not in either a sport or band.

You can use two-way tables to find **relative frequency.** Relative frequency = $\frac{\text{the number of times a condition is observed}}{\text{total number of observations}}$

If Lori flips a coin 100 times and it lands on heads 53 times, the relative frequency of the coin landing on heads to total coin tosses is $\frac{53}{100}$. **Example:** Use the two-way table to find the relative frequency of students who have both a cat and a dog to total students surveyed.

	cat	no cat	total
dog	15	20	35
no dog	41	24	65
total	56	44	100

Answer: The relative frequency is $\frac{15}{100}$. Looking at the table, you can see that the total number of people surveyed is 100. The denominator of the relative frequency will be 100. The table shows that there are 15 people who have both a cat and a dog. 15 is the numerator of the relative frequency.

Help Pages Index

Help Pages Index