Summer Solutions.

Minutes a Day–Mastery for a Lifetime!

Standards-Based Mathematics 5

Help Pages

Help Pages

Vocabulary

Acute angle — an angle measuring less than 90°

Area — the amount of space within a polygon; area is always measured in square units (feet^2, meters^2, etc.)

Axis / Axes — the lines that form the framework for a graph. The horizontal axis is called the *x*-axis; the vertical axis is called the *y*-axis.

Congruent — figures with the same shape and the same size

Coordinates – an ordered pair of numbers that give the location of a point in a coordinate grid

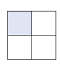
Coordinate Plane / Grid – a grid in which the location is described by its distances from two intersecting, straight lines called axes

Decimal — a number that contains a decimal point; any whole number or fraction can be written as a decimal; **Example:** $\frac{1}{10}$ = 0.10

Denominator — the bottom number of a fraction **Example**: $\frac{1}{4}$; the denominator is 4

Difference — the result or answer to a subtraction problem **Example**: The difference of 5 and 1 is 4.

Equivalent fractions — fractions with different names but equal value

Fraction — a part of a whole **Example**:  This box has 4 parts. 1 part is shaded. $\frac{1}{4}$ is shaded.

Isosceles triangle — a triangle that has two sides that are the same length

Mixed number — the sum of a whole number and a fraction **Example**: $5\frac{3}{4}$

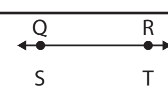
Numerator — the top number of a fraction. Example: $\frac{1}{4}$; the numerator is 1.

Obtuse angle — an angle measuring more than 90°

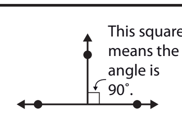
Ordered Pair – a pair of numbers that gives the coordinates of a point on a grid

Origin – the point where the x-axis and y-axis intersect

Parallel lines — two lines that never intersect and are always the same distance apart



Perpendicular lines — lines that intersect and form a right angle (90°)



Product — the result or answer to a multiplication problem **Example**: The product of 5 and 3 is 15.

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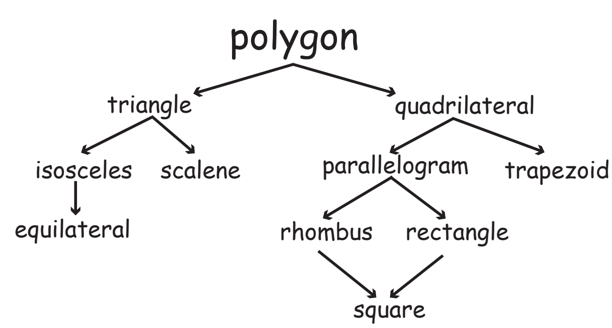
Vocabulary

Quotient — the result or answer to a division problem **Example**: The quotient of 8 and 2 is 4.
Right angle — an angle measuring exactly 90°
Right triangle — a triangle with one angle that measures exactly 90°
Scalene triangle — a triangle where no sides have the same length
Sum — the result or answer to an addition problem **Example**: The sum of 5 and 2 is 7.
Unit fraction — a fraction with a numerator of 1
Volume – the number of cubic units it takes to fill a solid; volume is expressed in cubic units (ft^3, m^3, $in.^3$)

Geometry — Polygons

Number of Sides	Name	Number of Sides	Name
3	Triangle	4	Quadrilateral
3	Right Triangle	5	Pentagon
3	Isosceles Triangle	6	Hexagon
3	Scalene Triangle		

Geometry — Classification of Polygons



Geometry — Classification of Triangles

Triangles can be classified by <u>sides</u> and <u>angles</u>.

Sides:

All the sides of an **equilateral triangle** are the same length.

At least two sides of an **isosceles triangle** are the same length.

No sides of a **scalene triangle** are the same length. A scalene triangle can be right, acute, or obtuse.

Angles:

A **right triangle** has one angle that measures 90°. It can be both scalene and isosceles, but not equilateral.

An **acute triangle** has three angles that measure between 0° and 90°.

An **obtuse triangle** has only one angle that measures greater than 90° and less than 180°.

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Measurement — Equivalent Units

Volume	Distance
1 liter (L) = 1,000 milliliters (mL)	1 foot (ft) = 12 inches (in.)
1 pint (pt) = 2 cups (C)	1 yard (yd) = 3 foot (ft) = 36 inches (in.)
1 gallon (gal) = 4 quarts (qt)	1 meter (m) = 100 centimeters (cm)
1 tablespoon (tbsp) = 3 teaspoons (tsp)	1 kilometer (km) = 1,000 meters (m)
Weight	**Time**
1 kilogram (kg) = 1,000 grams (g)	1 hour (hr) = 60 minutes (min)
1 pound (lb) = 16 ounces (oz)	1 minute (min) = 60 seconds (sec)

Whole Numbers — Decimals

1,	2	7	1,	4	0	5	.	6	4	9
millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	*decimal point* ⇧	tenths	hundredths	thousandths

The number above is read:
one million, two hundred seventy one thousand, four hundred five and six hundred forty-nine thousandths

Whole Numbers — Rounding to Any Place Value

When we **round numbers**, we are estimating them. This means we focus on a particular place value and decide if that digit is closer to the next highest number (round up) or to the next lower number (keep the same). It might be helpful to look at the place value chart above.

Example: Round 4,826 to the hundreds place.

rounding place → 4,<u>8</u>26

4,8<u>2</u>6

Since 2 is less than 5, the rounding place <u>stays the same</u>.

4,800

1. Identify the place that you want to round to. What number is in that place? (8)
2. Look at the digit to its right.
3. If this digit is 5 or greater, increase the number in the rounding place by 1. (round up) If the digit is less than 5, keep the number in the rounding place the same.
4. Replace all digits to the right of the rounding place with zeros.

Example: Round 27,934 to the thousands place.

2<u>7</u>,934 ⟶ 7 is in the rounding place.
27,<u>9</u>34 ⟶ 9 is greater than 5, so the rounding place will go up by 1.
28,000 ⟶ The digits to the right of the rounding place are changed to zeros.

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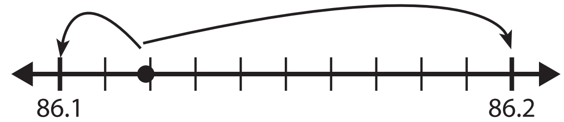
Rounding Decimals

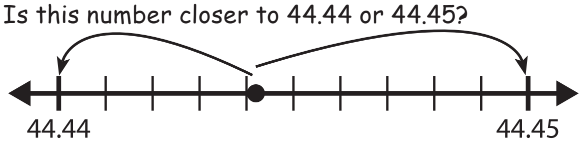
When we round decimals, we are approximating them. This means we end the decimal at a certain place value and we decide if it's closer to the next higher number (round up) or to the next lower number (keep the same).

Example 1:
Round 86.119 to the nearest tenth.
Is this number closer to 86.1 or 86.2?



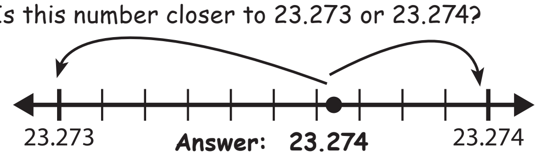
Answer: 86.1

Example 2:
Round 44.444 to the nearest hundredth.
Is this number closer to 44.44 or 44.45?



Answer: 44.44

Example 3:
Round 23.2736 to the nearest thousandth.
Is this number closer to 23.273 or 23.274?



Answer: 23.274

Example 4: Round 0.574 to the tenths place.

There is a 5 in the rounding (tenths) place. — **0.<u>5</u>74**

Since 7 is greater than 5, change the 5 to a 6. — **0.<u>5</u>74**

Drop the digits to the right of the tenths place. — **0.6**

1. Identify the number in the rounding place.
2. Look at the digit to its right.
3. If the digit is 5 or greater, increase the number in the rounding place by 1. If the digit is less than 5, keep the number in the rounding place the same.
4. Drop all digits to the right of the rounding place.

Example 5: Round 2.783 to the nearest hundredth.

2.7<u>8</u>3 — There is an 8 in the rounding place.

2.7<u>8</u>3 — Since 3 is less than 5, keep the rounding place the same.

2.78 — Drop the digits to the right of the hundredths place.

Expanded Notation — (Decimals)

Base-Ten	Expanded Form	Word
0.45	$(4 \times \frac{1}{10}) + (5 \times \frac{1}{100})$	forty-five hundredths
15.137	$(1 \times 10) + (5 \times 1) + (1 \times \frac{1}{10}) + (3 \times \frac{1}{100}) + (7 \times \frac{1}{1{,}000})$	fifteen and one hundred thirty-seven thousandths
3.286	$(3 \times 1) + (2 \times \frac{1}{10}) + (8 \times \frac{1}{100}) + (6 \times \frac{1}{1{,}000})$	three and two hundred eighty-six thousandths
26.4	$(20 \times 10) + (6 \times 1) + (4 \times \frac{1}{10})$	twenty-six and four tenths
487.391	$(4 \times 100) + (8 \times 10) + (7 \times 1) + (3 \times \frac{1}{10}) + (9 \times \frac{1}{100}) + (1 \times \frac{1}{1{,}000})$	four hundred eighty-seven and three hundred ninety-one thousandths

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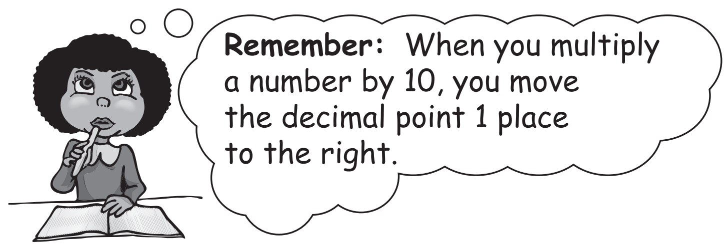
Powers of 10

$10^1 = 10 \times 1 = 10$
$10^2 = 10 \times 10 = 100$
$10^3 = 10 \times 10 \times 10 = 1,000$
$10^4 = 10 \times 10 \times 10 \times 10 = 10,000$
$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100,000$
$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000$

Multiply and Divide by a Power of 10 — Patterns

Multiply 18.735 by 10^1, 10^2, and 10^3.

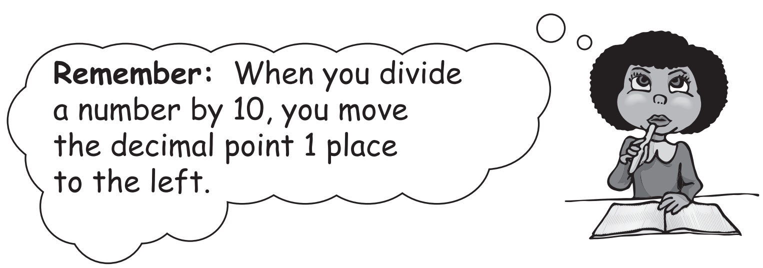


$18.735 \times 10^1 = 187.35$ The decimal point moves one place to the right.

$18.735 \times 10^2 = 1873.5$ The decimal point moves two places to the right.

$18.735 \times 10^3 = 18,735$ The decimal point moves three places to the right.

Divide 18.735 by 10^1, 10^2, and 10^3.



$18.735 \div 10^1 = 1.8735$ The decimal point moves one place to the left.

$18.735 \div 10^2 = 0.18735$ The decimal point moves two places to the left.

$18.735 \div 10^3 = 0.018735$ The decimal point moves three places to the left. A zero must be added.

Help Pages

Whole Numbers — Multiplication

When multiplying multi-digit whole numbers, it is important to know your multiplication facts. Follow the steps and the examples below.

Here is a way to multiply a four-digit whole number by a one-digit whole number.

Use the **distributive property** to multiply 3,514 x 3.

Multiply 3 by all the values in 3,514 (3,000 + 500 + 10 + 4).

Add all the partial products to get one final product.

$$\begin{array}{r} {}^{1}\ {}^{1}\quad \\ 3{,}514 \\ \times\quad 3 \\ \hline 10{,}542 \end{array}$$

3 × 4 = 12 ones or 1 ten and 2 ones.
3 × 10 = 3 tens + 1 ten (regrouped) or 4 tens.
3 × 500 = 15 hundreds or 1 thousand and 5 hundreds.
3 × 3,000 = 9 thousands + 1 thousand (regrouped) or 10 thousands.
(3,000 x 3) + (500 x 3) + (10 x 3) + (4 x 3) = 9,000 + 1,500 + 30 + 12 = 10,542.

Here are two ways to multiply two two-digit numbers.

Use the **distributive property** to multiply 36 x 12.

Multiply the two addends of 36 (30 + 6) by the two addends of 12 (10 + 2).

Then, add all the partial products to get one final product.

$$\begin{array}{r} 36 \\ \times\ 12 \\ \hline 432 \end{array}$$

2 x 6 = 12 2 x 30 = 60
10 x 6 = 60 10 x 30 = 300
(30 x 10) + (30 x 2) + (6 x 10) + (6 x 2) =
300 + 60 + 60 + 12 = **432**

Use the **matrix model** to multipy 48 x 31.

The model shows the four parts needed to arrive at the final product.

Place the expanded form of each two-digit number on the outside edge of the boxes as shown.

Write the partial products in each box. The sum of the four partial products is 1,488.

Notice the two different addition problems that serve as a way to check your accuracy.

	40	8	
30	1,200	240	1,440
1	40	8	+ 48
	1,240 +	248	1,488

Factors and Multiples

In the basic fact 2 × 3 = 6. 2 and 3 are called **factors,** and the **product** is 6.

To name all the **factor pairs** of 20, think of every factor pair that will result in a product of 20 (1 x 20, 2 x 10, 4 x 5). Then list those factors from smallest to largest (1, 2, 4, 5, 10, and 20).

A **multiple** is the product of two whole numbers. When you skip count by twos, you say the multiples of two. The first five multiples of 2 are 2, 4, 6, 8, and 10.

Prime and Composite

Prime Numbers: A prime number is a number greater than 1 that has only two factors, 1 and itself. 2 and 7 are prime numbers: 2 × 1 = 2; 7 × 1 = 7.

Composite Numbers: A composite number has more than two factors. 12 is a composite number with 6 factors: 1, 2, 3, 4, 6, 12.

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Whole Numbers — Division

This example involves **division using two-digit divisors with remainders**. You already know how to divide single-digit numbers. This process, called "long division," helps you divide numbers with multiple digits.

Example: Divide 6,392 by 27.

1. There are 200 twenty-sevens in 6,392.
   The 2 is in the hundreds place.

$$\begin{array}{r} 2 \\ 27\overline{)6,392} \end{array}$$

2. 200 times twenty-seven equals 5,400.
   6,392 minus 5,400 equals 992.

$$\begin{array}{r} 2 \\ 27\overline{)\ 6,392} \\ \underline{-5,400} \\ 992 \end{array}$$

3. There are 30 twenty-sevens in 992.
   The 3 is in the tens place.

$$\begin{array}{r} 23 \\ 27\overline{)\ 6,392} \\ \underline{-5,400} \\ 992 \end{array}$$

4. 30 times twenty-seven equals 810.
   992 minus 810 equals 182.

$$\begin{array}{r} 23 \\ 27\overline{)\ 6,392} \\ \underline{-5,400} \\ 992 \\ \underline{-810} \\ 182 \end{array}$$

5. There are 6 twenty-sevens in 182.
   The six is in the ones place.

$$\begin{array}{r} 236 \\ 27\overline{)\ 6,392} \\ \underline{-5,400} \\ 992 \\ \underline{-810} \\ 182 \end{array}$$

6. 6 times twenty-seven equals 162.
   182 minus 162 equals 20.
   20 is less than 27, so the remainder is 20.

$$\begin{array}{r} 236 \\ 27\overline{)\ 6,392} \\ \underline{-5,400} \\ 992 \\ \underline{-810} \\ 182 \\ \underline{-162} \\ 20 \end{array}$$

The quotient is 236, R20 or $236\frac{20}{27}$.

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Whole Numbers — Division - Place Value Model

Example: Solve. 8, 524 ÷ 4

1. Expand the dividend and write it in the place value model. 8,524 = 8,000 + 500 + 20 + 4
   The dotted box is for a remainder.
2. Put the divisor in front of the model.
3. Divide 4 into each place value.

4	8,000	500	20	4	

   - How many 4s are in 8,000? (2,000 x 4 = 8,000)

	2,000				
4	8,000	500	20	4	

   - How many 4s are in 500? (125 x 4 = 500)

	2,000	125			
4	8,000	500	20	4	

   - How many 4s are in 20? (5 x 4 = 20)

	2,000	125	5		
4	8,000	500	20	4	

   - How many 4s are in 4? (1 x 4 = 4)

	2,000	125	5	1	
4	8,000	500	20	4	

4. Record the partial quotients.
5. Add the numbers in step 4 to find the final quotient.

$$\begin{array}{r} 2{,}000 \\ 125 \\ 5 \\ +\ 1 \\ \hline 2{,}131 \end{array}$$

8,524 ÷ 4 = **2,131**. There is no remainder.

Example: Solve. 457 ÷ 2

1. Expand the dividend and write it in the place value model. 457 = 400 + 50 + 7
   The dotted box is for a remainder.
2. Put the divisor in front of the model.
3. Divide 2 into each place value.

2	400	50	7	

   - How many 2s are in 400? (200 x 2 = 400)

	200			
2	400	50	7	

   - How many 2s are in 50? (25 x 2 = 50)

	200	25		
2	400	50	7	

   - How many 2s are in 7? (3 x 2 = 6)

	200	25	3	
2	400	50	7	1

4. Record the partial quotients.
5. Add the numbers in step 4 to find the final quotient.

	200	25	3	
2	400	50	7	1

$$\begin{array}{r} 200 \\ 25 \\ +\ 3 \\ \hline 228 \end{array}$$

457 ÷ 2 = 228 (with a remainder of 1) or **228 R1**

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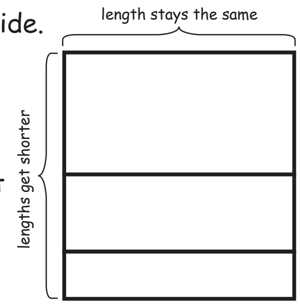
Whole Numbers — Division — Area Model

An **area model** can help you keep track of how much of the dividend is left to divide.

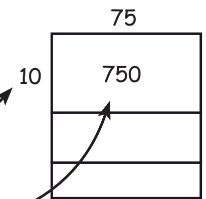
The sections of the model represent chunks of a divisor that you subtract from the dividend. The chunks are shaped like rectangles to help you imagine different-sized areas that have different side lengths. The side length of the divisor stays the same, but the other side length gets shorter as the amount left in the dividend gets smaller.



Example: Follow these steps to solve 1,350 ÷ 75.

1. Put the divisor (75) above the area model. Think about friendly numbers that are easy to multiply by 75. There are at least 10 groups of 75 in 1,350, so start with 10.

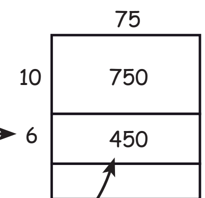
   Write 10 to the left of the rectangle. Multiply 10 x 75.
   The product is 750. In the model, write 750 in the top rectangle.
   Off to the side, show that 1,350 – 750 = 600.



$$\begin{array}{r} 1{,}350 \\ -750 \\ \hline 600 \end{array}$$

2. Next, decide how many groups of 75 are in 600. Since 75 is very close to 100, there have to be at least 6 groups of 75 in 600.

   Write 6 to the left of the rectangle. Multiply 6 x 75.
   The product is 450. In the model, write 450 in the next rectangle.
   Off to the side, show that 600 – 450 = 150.

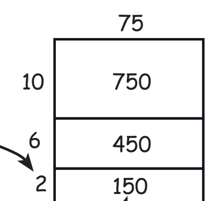


$$\begin{array}{r} 1{,}350 \\ -750 \\ \hline 600 \\ -450 \\ \hline 150 \end{array}$$

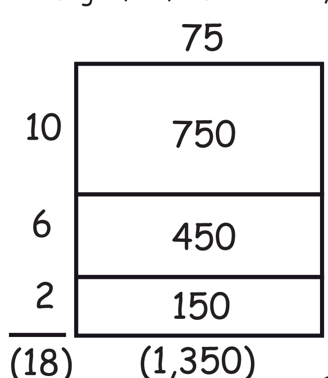
3. Finally, decide how many groups of 75 are left in 150. There are 2.

   Write 2 to the left of the rectangle. Multiply 2 x 75.
   The product is 150. In the model, write 150 in the bottom rectangle.
   Off to the side, show that 150 – 150 = 0.

   There is nothing left of the dividend, so your work there is complete.



$$\begin{array}{r} 1{,}350 \\ -750 \\ \hline 600 \\ -450 \\ \hline 150 \\ -150 \\ \hline 0 \end{array}$$



4. The three rectangles together represent the whole dividend:
   750 + 450 + 150 = 1,350.

5. Add the numbers at the left of the rectangles.
   This sum represents the quotient: 10 + 6 + 2 = 18.

6. 1,350 ÷ 75 = 18

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Whole Numbers — Division using Estimation and Equations

Example: Solve. 731 ÷ 25 = *n*

A related multiplication equation is 25 × *n* = 731.

Estimation:

If (4 × 25) = 100, then 7 × (4 × 25) = 700; (7 × 4) × 25 = 28 × 25 = 700
Also, 3 × 25 = 75, so 25 × 3 tens (30) = 75 tens or 750.

731 ÷ 25 = 29 R6

My answer should be greater than 28 but less than 30.

Decimals — Addition

Example: Solve. 5.2 + 3.9 = __

5.2 is close to 5. 3.9 is close to 4.
Since 5 + 4 = 9, the sum should be about 9.

```
  1
  5.2
+3.9
  9.1
```

Line up the numbers with the same place value, then add.

Decimals — Subtraction

Example: Solve. 8.3 − 2.7 = ___

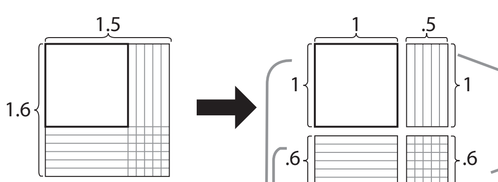
When the numbers are estimated to the nearest whole number, the problem becomes 8 − 3.
Since 8 − 3 = 5, the sum should be about 5.

```
  7 1
  8.3
−2.7
  5.6
```

Line up the numbers with the same place value, then subtract.

Decimals — Multiplication

Example: Solve. 1.5 x 1.6 = ___



Think of this model as 4 separate rectangles.

Find the area of each, then add.

(1 x 1) = 1.00
(1 x .6) = 0.60
(1 x .5) = 0.50
(.5 x .6) = 0.30
2.40

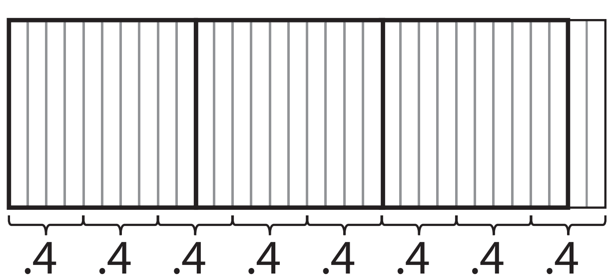
```
  1.5
×1.6
 2.40
```

Decimals — Division

Example: Solve. 3.2 ÷ 0.4 = ___

The model represents 3 ones and 2 tenths. Each one is partitioned into 10 equal sections representing 10 tenths. The model shows how many groups of 4 tenths are in 3.2.

Count the 8 groups. 3.2 ÷ 0.4 = 8



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Fractions — Equivalent Fractions

Equivalent Fractions are 2 fractions that are equal to each other. Usually you will be finding a missing numerator or denominator.

Example: Find a fraction that is equivalent to $\frac{4}{5}$ and has a denominator of 35.

× 7

$\frac{4}{5} = \frac{?}{35}$

× 7

1. Ask yourself, "What did I do to 5 to get 35?" (Multiplied by 7.)
2. Whatever you did in the denominator, you also must do in the numerator. 4 x 7 = 28 The missing numerator is 28.

So, $\frac{4}{5}$ is equivalent to $\frac{28}{35}$.

Example: Find a fraction that is equivalent to $\frac{4}{5}$ and has a numerator of 24.

× 6

$\frac{4}{5} = \frac{24}{?}$

× 6

1. Ask yourself, "What did I do to 4 to get 24?" (Multiplied by 6.)
2. Whatever you did in the numerator, you also must do in the denominator. 5 x 6 = 30 The missing denominator is 30.

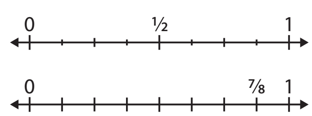
So, $\frac{4}{5}$ is equivalent to $\frac{24}{30}$.

Fractions — Comparing Fractions

When you are given a visual model like a number line to compare two fractions, the fraction that is farthest to the right on the number line is greater.

Example: Choose the sign that makes this sentence true. (< > =) $\frac{1}{2} \bigcirc \frac{7}{8}$

Find the fractions on each number line. The fraction that is farthest to the right is greater.



$\frac{1}{2} < \frac{7}{8}$

When you do not have a number line, think about what you already know.

Example: Choose the sign that makes this sentence true. $\frac{3}{8} \bigcirc \frac{1}{2}$

Think about this: Half of 8 is 4, so $\frac{4}{8}$ equals $\frac{1}{2}$. Which has more eighths, $\frac{3}{8}$ or $\frac{4}{8}$? That will help you know which is greater.

$$\frac{3}{8} < \frac{1}{2}$$

Example: Choose the sign that makes this sentence true. (< > =) $\frac{5}{6} \bigcirc \frac{2}{6}$

To compare fractions with like denominators, simply compare the numerators.

$\frac{5}{6} > \frac{2}{6}$, because 5 > 2.

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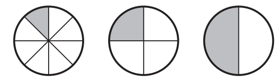
Fractions — Comparing Fractions (continued)

Compare the unit fractions at the right. Notice that the larger the denominator, the smaller the unit is.



$\frac{1}{8} < \frac{1}{4} < \frac{1}{2}$

Example: Choose the sign that makes this sentence true. (< > =) $\frac{4}{5} \bigcirc \frac{4}{10}$

To compare fractions with like numerators, remember that the larger the denominator, the smaller the unit is.

$\frac{4}{5} > \frac{4}{10}$ This is true because fifths are larger units than tenths are.

Fractions — Adding and Subtracting with Unlike Denominators

To add or subtract fractions, the fractions must have a common denominator.

Examples: $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$ $\frac{7}{12} - \frac{5}{12} = \frac{2}{12}$

If the denominators are the same, simply add or subtract to find the sum or difference.

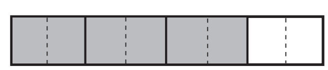
For unlike denominators, you must find a common denominator. Follow these steps to add $\frac{1}{2}$ and $\frac{1}{4}$:

$\frac{1}{2} + \frac{1}{4} =$

$\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$ and $\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$

$\frac{4}{8}$ is equivalent to $\frac{1}{2}$ $\frac{2}{8}$ is equivalent to $\frac{1}{4}$

1. Multiply each fraction by the other fraction's denominator. When you do this, you are not changing the value of the fraction because you are multiplying by 1. (Any fraction with the same numerator and denominator is equal to 1.)
2. Now you can add the fractions. $\frac{4}{8} + \frac{2}{8} = \frac{6}{8}$

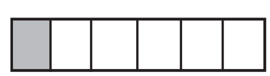


Follow these steps to find $\frac{2}{3} - \frac{1}{2}$:

$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$ and $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$

$\frac{4}{6}$ is equivalent to $\frac{2}{3}$ $\frac{3}{6}$ is equivalent to $\frac{1}{2}$

1. Multiply each fraction by the other fraction's denominator. When you do this, you are not changing the value of the fraction because you are multiplying by 1. (Any fraction with the same numerator and denominator is equal to 1.)
2. Now you can subtract the fractions. $\frac{4}{6} - \frac{3}{6} = \frac{1}{6}$



Help Pages

Fractions — Adding and Subtracting Mixed Numbers with Unlike Denominators

To add or subtract mixed numbers, the fractions must have a common denominator.
If the denominators are the same, simply add or subtract to find the sum or difference.

Examples: $7\frac{2}{9} + 3\frac{1}{9} = 10\frac{3}{9}$　　$(7 + 3 = 10 \text{ and } \frac{2}{9} + \frac{1}{9} = \frac{3}{9})$

$6\frac{3}{5} - 2\frac{2}{5} = 4\frac{1}{5}$　　$(6 - 2 = 4 \text{ and } \frac{3}{5} - \frac{2}{5} = \frac{1}{5})$

For mixed numbers with unlike denominators, follow these steps to find a common denominator.

Example: $42\frac{1}{3} + 4\frac{1}{4} = ?$

1. Add the whole numbers. 42 + 4 = 46
2. Follow the steps to find a common denominator: $\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$ and $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$
3. Add the fractions. $\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$, so the sum is $42\frac{7}{12}$.

Example: $19\frac{4}{5} - 12\frac{1}{8} = ?$

1. Follow the steps to find a common denominator: $\frac{4}{5} \times \frac{8}{8} = \frac{32}{40}$ and $\frac{1}{8} \times \frac{5}{5} = \frac{5}{40}$
2. Subtract the fractions. $\frac{32}{40} - \frac{5}{40} = \frac{27}{40}$; the difference is $\frac{27}{40}$.
3. Subtract the whole numbers. 19 – 12 = 7, so the difference is $7\frac{27}{40}$.

Fractions — Subtracting Mixed Numbers with Regrouping

Sometimes, you have to regroup when you subtract.

Example: $\begin{array}{r} 21\frac{1}{5} \\ -3\frac{2}{3} \\ \hline \end{array}$　You cannot subtract $\frac{2}{3}$ from $\frac{1}{5}$.　$\begin{array}{r} 21\frac{3}{15} \\ -3\frac{10}{15} \\ \hline \end{array}$　Find a common denominator. You still cannot subtract $\frac{10}{15}$ from $\frac{3}{15}$.

Follow these steps:

1. Find a common denominator for $\frac{1}{5}$ and $\frac{2}{3}$.　$\frac{1}{5} \times \frac{3}{3} = \frac{3}{15}$ and $\frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$.
2. Set up the equation as shown below. You cannot subtract $\frac{10}{15}$ from $\frac{3}{15}$, so take 1 from 21 (make it 20) and then add 1 (in the form of $\frac{15}{15}$) to the fraction to get $\frac{18}{15}$.
   This works because you are not changing the value of the mixed number; you are only renaming it.

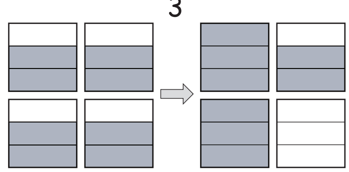
$$\begin{array}{r} 21\frac{1}{5} \\ -3\frac{2}{3} \\ \hline \end{array} \qquad \begin{array}{r} \not{21}\frac{3+15}{15} \\ -3\ \frac{10}{15} \\ \hline \end{array} \quad \begin{array}{c} = \\ = \end{array} \quad \begin{array}{r} 20\frac{18}{15} \\ -3\frac{10}{15} \\ \hline \end{array}$$

3. Subtract the whole numbers: 20 – 3 = 17. Then, subtract the fractions: $\frac{18}{15} - \frac{10}{15} = \frac{8}{15}$
   The difference is $17\frac{8}{15}$.

Fraction Models for Multiplication

Example: Use the fraction model to solve $4 \times \frac{2}{3}$. This fraction model shows $4 \times \frac{2}{3}$.

The first model shows that four groups of $\frac{2}{3}$ is $\frac{8}{3}$.



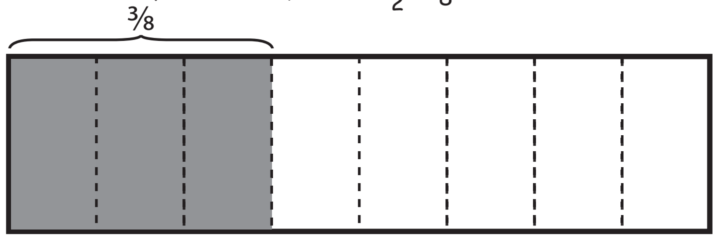
The second model shows that $\frac{8}{3}$ is equal to the mixed number $2\frac{2}{3}$.

Help Pages

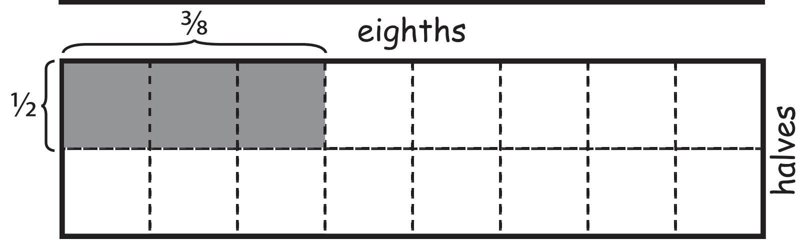
Fractions — Multiplying

What is $\frac{1}{2}$ of $\frac{3}{8}$? To find out, write a multiplication equation: $\frac{1}{2} \times \frac{3}{8} = ?$

This model shows $\frac{3}{8}$.



This model shows $\frac{1}{2}$ of $\frac{3}{8}$.



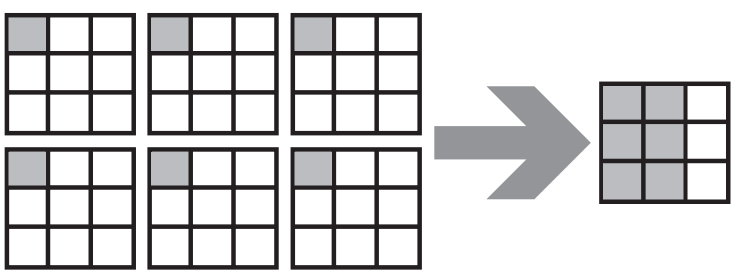
You can see that $\frac{1}{2}$ of $\frac{3}{8}$, or $\frac{1}{2}$ times $\frac{3}{8}$, is $\frac{3}{16}$. (When you see the word *of*, it usually means you will multiply.)

To find the product of two fractions, multiply the numerator by the numerator and the denominator by the denominator.

Examples: $\frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$ $\frac{2}{5} \times \frac{1}{2} = \frac{2}{10}$ $\frac{3}{4} \times \frac{1}{9} = \frac{3}{36}$

Fractions — Multiplying a Fraction by a Whole Number

What is $6 \times \frac{1}{9}$? Study the fraction model. It shows 6 one-ninths or $\frac{6}{9}$.



Every whole number can be written as itself over 1. For example, 6 is the same as $\frac{6}{1}$ because $\frac{6}{1}$ means $6 \div 1$ and that equals 6.

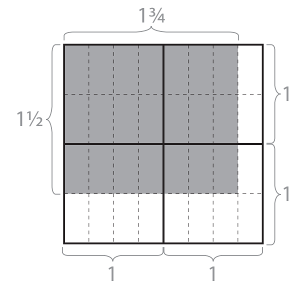
To multiply a fraction by a whole number, show the whole number in its fraction form and multiply the numerators and denominators. (Remember, any whole number can be expressed as a fraction; the whole number becomes the numerator, and 1 is the denominator. This works because any number divided by 1 is that number.)

Examples: $7 \times \frac{1}{7} = \frac{7}{1} \times \frac{1}{7} = \frac{7}{7} = 1$ $\frac{1}{3} \times 15 = \frac{1}{3} \times \frac{15}{1} = \frac{15}{3} = 5$

$8 \times \frac{11}{12} = \frac{8}{1} \times \frac{11}{12} = \frac{88}{12} = 7\frac{4}{12} = 7\frac{1}{3}$

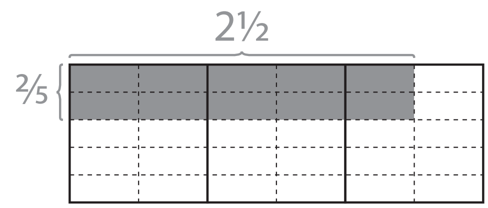
Help Pages

Fractions — Multiplying Mixed Numbers



Study the fraction model. It shows $1\frac{1}{2}$ sets of $1\frac{3}{4}$.

The model shows that $1\frac{1}{2}$ of $1\frac{3}{4}$ is $2\frac{5}{8}$. Every 8 subsections make one whole. So, count 8 subsections and 8 more. That makes 2 whole squares. There are 5 subsections left over, so that makes $2\frac{5}{8}$.



What is $\frac{2}{5}$ of $2\frac{1}{2}$? Study the fraction model.
This model shows that $2\frac{1}{2}$ is 5 halves.

2 of the 5 halves equal 1 whole rectangle, so $\frac{2}{5}$ of $2\frac{1}{2}$ is 1.
To check this, study the next section entitled "Fractions - Converting a Mixed Number to an Improper Fraction."

Fractions — Converting a Mixed Number to an Improper Fraction

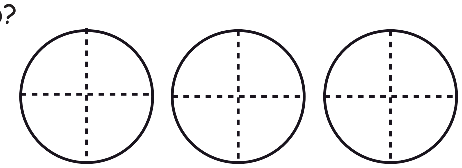
Multiply $\frac{2}{5} \times 2\frac{1}{2}$. First, you must convert the mixed number ($2\frac{1}{2}$) to an improper fraction.

- Multiply the whole number by the denominator. 2 x 2 = 4
- Add the numerator. 4 + 1 = 5
- Use that sum as the new numerator and keep the denominator. The improper fraction is $\frac{5}{2}$.

$$\frac{2}{5} \times \frac{5}{2} = \frac{10}{10} = 1$$

Fractions — Dividing Whole Numbers by Fractions

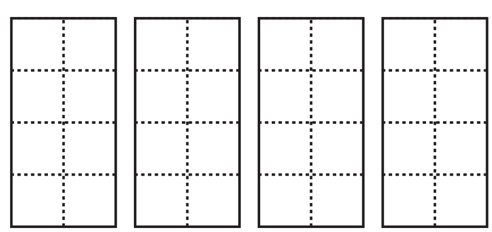
A fifth grade class has 3 hours of time in the computer lab. If the teacher spends $\frac{1}{4}$ of an hour on each activity, how many activities will the class be able to do?



Use the fraction model to find the quotient of $3 \div \frac{1}{4}$.

The model shows that there are 12 one-fourths in 3. Three whole hours each have 4 quarter hours, and 3 x 4 = 12 quarter hours. Notice that you can get the same answer by multiplying the whole number by the fraction's denominator. This is called multiplying by the inverse.

$3 \div \frac{1}{4} = 12$ is the same as $\frac{3}{1} \times \frac{4}{1} = \frac{12}{1} = 12$



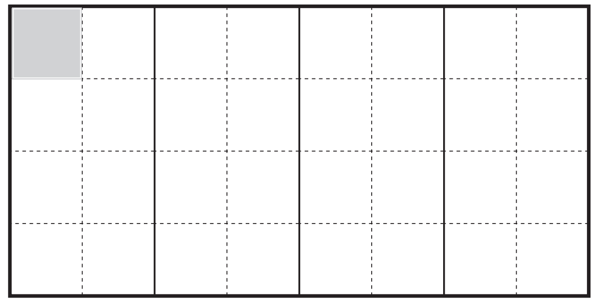
What is $4 \div \frac{1}{8}$? Study the fraction model.
It shows that there are 32 one-eighths in 4.

To check the quotient of $4 \div \frac{1}{8}$, multiply 4 by the inverse of $\frac{1}{8}$: $\frac{4}{1} \times \frac{8}{1} = \frac{32}{1} = 32$

Help Pages

Fractions — Dividing Fractions by Whole Numbers

Mrs. Marx cut a sheet of poster board into equal pieces for 4 groups of students. If there are 8 students in each group, and the students in each group share equally, what fraction of the whole poster board will each student get?

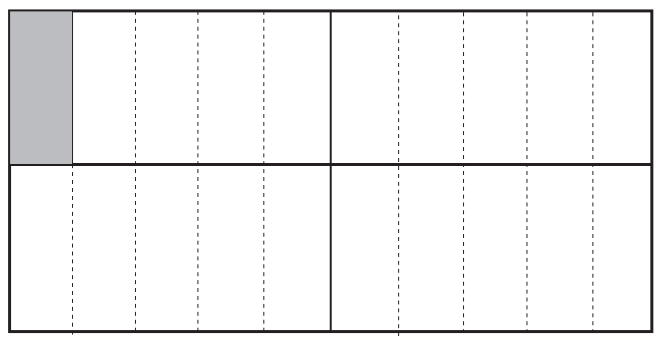


Use a fraction model to find $\frac{1}{4} \div 8$.

The fraction model shows that if each $\frac{1}{4}$ of the poster board is equally divided into 8 sections, each student will get $\frac{1}{32}$ of the original poster board.

Notice that you can get the same answer by multiplying the fraction's denominator by the inverse of the whole number.

$$\frac{1}{4} \div \frac{8}{1} = \frac{1}{32} \text{ is the same as } \frac{1}{4} \times \frac{1}{8} = \frac{1}{32}$$



What is $\frac{1}{4} \div 5$? Study the fraction model.
It shows that $\frac{1}{4} \div 5 = \frac{1}{20}$.

To check the quotient, multiply by the inverse.

$$\frac{1}{4} \div \frac{5}{1} = \frac{1}{20} \text{ is the same as } \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$$

Fractions and Decimals

All fractions with denominators of 10 and 100 can be written as decimals. The decimal 0.50 can be described as 5 tenths or 50 hundredths.

$$\frac{5}{10} = 0.5$$

$$\frac{50}{100} = 0.50$$

The fraction and decimal below can be decomposed (broken down) into $0.5\left(\frac{5}{10}\right) + 0.03\left(\frac{3}{100}\right)$.

$$\frac{53}{100} = 0.53$$

Help Pages

Solve Expressions Using Parentheses ( ), Brackets [ ], and Braces { }

Look at the expression {64 ÷ [2 × (10 – 6)]} – 2. When solving algebraic expressions, work with the innermost groupings first.

1. Solve what's inside the parentheses first. (10 – 6) = 4
2. Solve what's inside the brackets next. {64 ÷ [2 × 4]} – 2
3. Solve what's inside the braces last. {64 ÷ 8} – 2

Solution : **8 – 2 = 6**

Write Numerical Expressions

Sometimes you are given a numerical expression using words and numbers.

Example 1: Add 12 and 24, then divide by 6. (Divide the phrase into parts and then combine the parts.)

Part I – add 12 and 24 12 + 24
Part II – then divide by 6 ÷ 6

Combine the parts and add parentheses. **(12 + 24) ÷ 6**

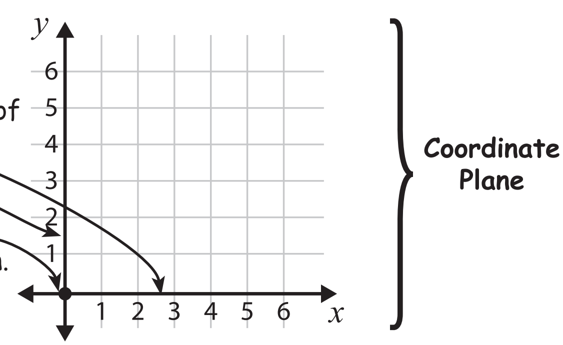
Example 2: 9 multiplied by 8, then subtract 22.

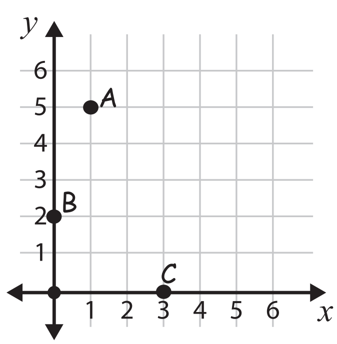
Part I 9 multiplied by 8 = 9 x 8
Part II subtract 22 = – 22

Combine the parts and add parentheses. **(9 x 8) – 22**

Graphing on a Coordinate Plane

A **coordinate plane** is formed by the intersection of a horizontal number line, called the **x–axis**, and a vertical number line, called the **y–axis**. The axes meet at the point (0, 0), called the **origin**.





Points are shown by **ordered pairs** of numbers, (x, y). The first number in an ordered pair is the **x coordinate**; the second number is the **y-coordinate**. In the point (3, 2), **3** is the x-coordinate and **2** is the y-coordinate. When graphing on a coordinate plane, always move on the x-axis first (right) and then move on the y-axis (up).

The coordinates of point **A** are **(1, 5)**.
The coordinates of point **B** are **(0, 2)**.
The coordinates of point **C** are **(3, 0)**.

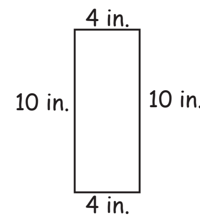
Help Pages

Rectangles — Perimeter

A rectangle has 2 pairs of parallel sides. The distance around the outside of a rectangle is the perimeter. To find the perimeter of a rectangle, add the lengths of the sides.

Example:

$10 + 4 + 10 + 4 = 28$ in.
or
$2\ (10 + 4) = 2 \times 14 = 28$ in.
or
$(2 \times 10) + (2 \times 4) = 28$ in.



Rectangles — Area

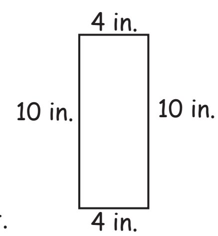
Area is the number of square units within any two-dimensional shape.

A rectangle has side lengths called length and width.

To find the area of a rectangle, multiply the length by the width ($l \times w$).

In the example, $10 \times 4 = 40$ in.^{2}

If the area is known, but the length or width is missing, use division to find the missing measurement.



Remember to label your answer in square units. **Examples:**
- square inches: in.^{2}
- square feet: ft^2
- square yards: yd^2
- square miles: mi^2
- square centimeters: cm^2
- square meters: m^2

Example: The area of a rectangle is 70 square inches. The length of one of the sides is 10 inches. Find the width Label the answer.

If $A = l \times w$, then $A \div w = l$ and $A \div l = w$. Show: $70 \div 10 = 7$.

The width is 7 inches.

Rectangles — Find the Length and Width

Example: The area of a rectangular sandbox is 18 m^2. The border around the sandbox is 18 m. What are the length and width of the sandbox? Use the factor pairs of 18 to help you.

In this example, the area and perimeter are clues to the size of the length and width. The area is 18 m^2. The factor pairs of 18 are 1 x 18, 2 x 9, and 3 x 6. One of those pairs can be the length and width of a rectangle that has a perimeter of 18 m. Use the guess and check strategy to find the right pair.

$1 + 1 + 18 + 18 \neq 18$
$2 + 2 + 9 + 9 \neq 18$
$3 + 3 + 6 + 6 = 18$

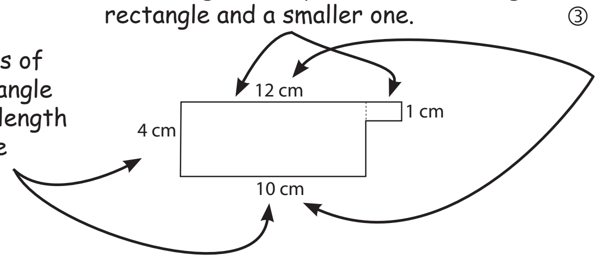
The length of the rectangle is 6 m and the width is 3 m.

Rectangles — Find the Area of Irregular Shapes

Example: Find the area of the shape. The dotted line helps to show two different rectangles. Find the area of each rectangle, and then add them together for a total.

① This irregular shape is made of a large rectangle and a smaller one.

② The side lengths of the larger rectangle are clear. The length is 10 cm and the width is 4 cm.



③ The small rectangle has a side length of 1 cm, but the other side is not labeled. However, notice that the top side length is 12 cm and the bottom one is 10 cm. By subtracting 10 from 12, you can see that the missing length is 2 cm. Use that number to calculate the smaller area.

(4×10) large rectangle + (2×1) small rectangle $= 40 + 2 = 42$ cm^2

The total area of the shape is 42 cm^2.

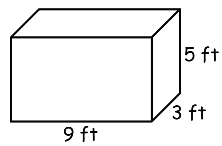
Help Pages

Volume

Volume is the measure of space inside of a solid figure. The volume of a rectangular prism is the product of its length, its width, and its height. Volume of a solid is expressed in cubic units (m^3, ft^3, etc.).

Volume = Length × Width × Height or V = L × W × H

Examples: Find the volume of the solids below.



Volume = Length × Width × Height

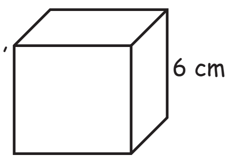
V = 9 ft × 3 ft × 5 ft

V = 135 ft^3 ➔ Say "135 cubic feet."

A cube has all sides equal, so its length, width, and height are all the same.

V = 6 cm × 6 cm × 6 cm

V = 216 cm^3



Interpreting Data — Line Plots

On a line plot you can quickly see data. It may be spread out or close together.

Example:

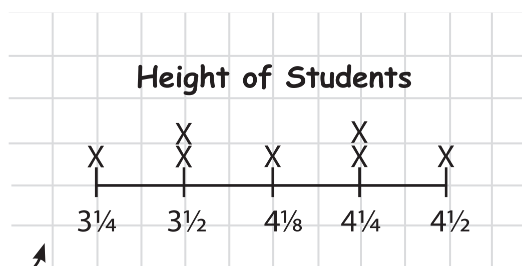
Gary recorded the heights of several students from different grades. Help Gary to organize his data into a line plot.

① To make a line plot, give the line plot a title.

Student	Height (ft)
Kelly	$3\frac{1}{4}$
Jerome	$4\frac{1}{2}$
Ming	$4\frac{1}{8}$
D'Andre	$4\frac{1}{4}$
Kyle	$3\frac{1}{2}$
Maria	$4\frac{1}{4}$
Hector	$3\frac{1}{2}$

② Find the greatest value and the lowest value in the set of data.

③ Draw a number line on the grid paper near the bottom. The number line should begin with the lowest value you found.



④ The length of your line should include space to mark from your lowest to your greatest value.

⑤ For each piece of data, draw an "x" above the matching value. An "x" on the line plot will take the place of each number from the data chart. No student names are needed.

What is the difference in height between the second tallest student and the shortest student?

The second tallest student is $4\frac{1}{4}$ ft. The shortest student is $3\frac{1}{4}$ ft.

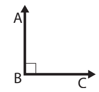
$$4\frac{1}{4} - 3\frac{1}{4} = 1 \text{ ft}$$

The difference between them is 1 foot.

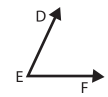
Help Pages

Geometric Measurement — Angles

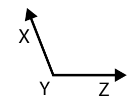
∠ABC is a right angle that measures exactly 90°.



∠DEF is an acute angle. An acute angle measures less than 90°.



∠XYZ is an obtuse angle. An obtuse angle measures greater than 90° and less than 180°.

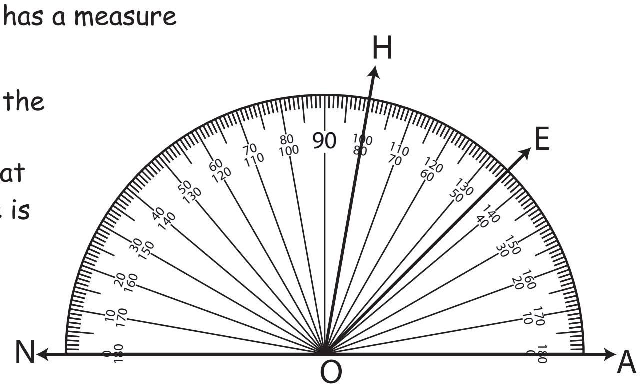


Geometric Measurement — Find the Measure of an Angle

To **find the measure of an angle**, a protractor is used.

The symbol for angle is ∠. On the diagram, ∠AOE has a measure less than 90°, so it is acute.

With the center of the protractor on the vertex of the angle (where the 2 rays meet), place one ray ($\overrightarrow{OA}$) on one of the "0" lines. Look at the number that the other ray ($\overrightarrow{OE}$) passes through. Since the angle is acute, use the lower set of numbers. Since $\overrightarrow{OE}$ is halfway between the 40 and the 50, the measure of ∠AOE is 45°. (If it were an obtuse angle, the higher set of numbers would be used.)

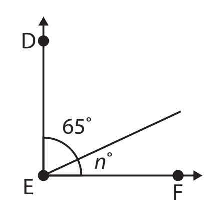


Look at ∠NOH. It is an obtuse angle, so the higher set of numbers will be used. Notice that $\overrightarrow{ON}$ is on the "0" line. $\overrightarrow{OH}$ passes through the 100 mark. So the measure of ∠NOH is 100°.

Geometric Measurement — Find the Missing Angle Measure

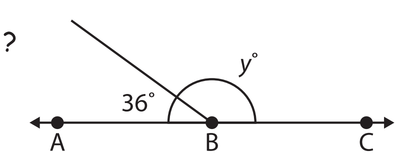
Example: If ∠DEF is a right angle (90°), what is the measure of *n* ?

In this example, 65 + *n* = 90. To find the missing angle measure *n*, subtract, 90 – 65 = *n*. The measure of *n* is 25°.



Example: If ∠ABC is a straight angle (180°), what is the measure of *y* ?

In this example, 36 + *y* = 180. To find the missing angle measure *y*, subtract 180 – 36 = *y*.
The measure of y is 144°.

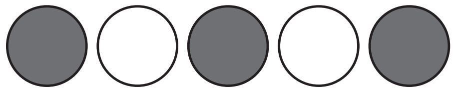


Help Pages

Describing Patterns

Patterns have shapes, numbers, or other items that either repeat or grow. The rule of a pattern describes HOW the pattern continues.

Study the pattern of shapes.



The rule of this pattern is the circles alternate color (gray, white, gray, white, gray).

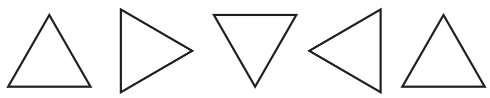
Study the pattern of numbers.

5, 10, 15, 20, 25

The rule of the pattern is "Start at 5 and add 5 each time."

Sometimes the pattern is easy to see, but the rule is harder to describe.

Here is a pattern of triangles.



The rule of the pattern is that the triangle rotates clockwise, 90 degrees each time.

Here is a pattern of numbers.

3, 13, 23, 33, 43

The rule of the pattern is "Start at 3 and add 10 each time."

A feature of a pattern is another way to describe the pattern.

Pattern	Rule	Feature
5, 10, 15, 20, 25	Start at 5; add 5.	All numbers end in 0 or 5.
3, 13, 23, 33, 43	Start at 3; add 10.	All numbers end in 3.
7, 9, 11, 13, 15, 17	Start at 7; add 2.	All numbers are odd.
3, 8, 13, 18, 23, 28	Start at 3; add 5.	All numbers end in 3 or 8.

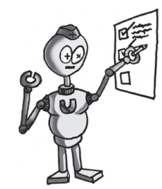
Help Pages

Problem Solving Strategies

Make an Organized List

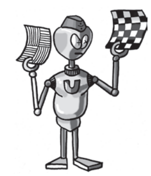
An **organized list** of possible answers for a problem uses an order that makes sense to you so that you do not miss any ideas or write the same answer more than once.

Guess and Check



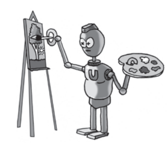
For the **guess and check** strategy, take a guess and see if it fits all the clues by checking each one. If it does, you have solved the problem. If it doesn't, keep trying until it works out. One way to know you have the best answer is when your answer fits <u>every</u> clue.

Look for a Pattern



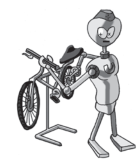
Sometimes math problems ask us to *continue a pattern by writing what comes next*. A **pattern** is an idea that repeats. In order to write what comes next in the pattern, you will first need to study the given information. As you study it, see if there is an idea that repeats.

Draw a Picture



When you **draw a picture** it helps you see the ideas you are trying to understand. The picture makes it easier to understand the words.

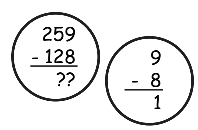
Work Backward



Using this strategy comes in handy when you know the end of a problem and the steps along the way, but you don't know how the problem began. If you start at the end and do the steps in reverse order you will end up at the beginning.

Solve a Simpler Problem

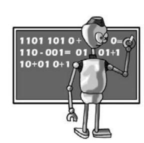
When you read a math problem with ideas that seem too big to understand, try to **solve a simpler problem**. Instead of giving up or skipping that problem, replace the harder numbers with easier ones.



Make a Table

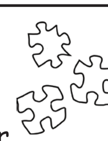
Tables have columns and rows. Labels are helpful too. Writing your ideas in this type of table (or chart) can help you organize the information in a problem so you can find an answer more easily. Sometimes it will make a pattern show up that you did not see before.

Write a Number Sentence



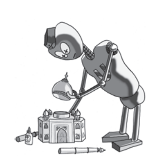
A **number sentence** is made up of numbers and math symbols (+ − × ÷ > < =). To use this strategy you will turn the words of a problem into numbers and symbols.

Use Logical Reasoning



Logical reasoning is basically common sense. **Logical** means "sensible." **Reasoning** is "a way of thinking." **Logical reasoning** is done one step at a time until you see the whole answer.

Make a Model



A **model** can be a picture you draw, or it can be an object you make or find to **help you understand the words** of a problem. These objects can be coins, paper clips, paper for folding, or cubes.